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UNIVERSITY OF ALBERTA

THE RECONSTRUCTION OF DECIMAL KNOWLEDGE IN YOUNG ADULTS

BY

YVETTE M. d'ENTREMONT ©

SUBMITTED TO THE FACULTY OF GRADUATE STUDIES AND
RESEARCH IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR
THE DEGREE OF DOCTOR OF PHILOSOPHY

DEPARTMENT OF SECONDARY EDUCATION

EDMONTON, ALBERTA
SPRING, 1991

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ABSTRACT

The purpose of this research was to ascertain the decimal number knowledge of Math 15 students, to study students' knowledge as it changed during a teaching experiment, to identify inter-relationships between held knowledge that the students possessed and knowledge growth under instruction, and to try to characterize such knowledge processes and inter-relationships theoretically. A pretest was used to ascertain the decimal number knowledge of the students and pretest student interviews served to identify the conceptual knowledge of the students.

Having identified aspects of decimal knowledge with which students appeared to have difficulty, a situation was designed where the students would come face-to-face with their misconceptions and where the teacher/researcher could observe what they did under those circumstances. This situation took the form of a teaching experiment. At the conclusion of the teaching experiment a post-test was administered. A delayed post-test was administered after a further ten weeks and delayed post-test interviews were carried out one month after the delayed post-test.

The analysis of the pretest and the pretest interviews indicated that the students who took part in the study possessed a hodgepodge of decimal knowledge. They used a variety of procedural techniques when solving decimal computation problems and tried to apply syntactic rules that they had previously been taught. Performance scores did increase after the teaching experiment and again at the delayed post-test. The interviews which took place one month after the delayed post-test suggested that the increased scores were due to a more complete and better organized procedural knowledge of decimals.

The actual growth in these students' decimal knowledge was analyzed by relating it to the Onion Model of Decimal Number Learning. The Onion Model is one metaphorical interpretation of how conceptual knowledge of decimal numbers can be attained. It consists of five layers: the concrete-objective layer, the operative-interpretive layer, the procedural layer, the mental model layer, and the abstract layer. The Onion Model suggests that conceptual knowledge is at the core of decimal knowledge and in order to reach the core one must peel away certain layers that surround the core. The results indicated that the students who took part in the study were at the procedural layer yet there existed differences in the students' ways of thinking about the procedures, their meaning and their use.

The metaphorical model suggested layers within layers. Within the procedural layer students differed in completeness and organization of knowledge and in the ability to observe patterns in procedural outcomes. These results suggested a reinterpretation of the model to include the possibility that some procedural knowledge could be conceptual in nature.

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CHAPTER I

The Problem

Introduction

The decimal number system is of great importance to the mathematical education of every student. The system provides mathematical models for a large number of the practical problems that students are likely to encounter. Yet research studies show that students perform poorly on tasks that require decimal computation and that most often the students apply a memorized computational rule in a meaningless way (Bell, Swan & Taylor, 1981; Hiebert & Wearne, 1984). Students apply memorized computational rules incorrectly without knowing why those rules are incorrect or that the answers are wrong.

The results of the National Assessment of Education Progress (Carpenter, Corbitt, Kepner, Lindquist & Reys, 1981) support the conclusion that students perform poorly on tasks involving decimal computation and that:

> Although performance increased about 20% from age 13 to age 17, many areas of difficulty for 13-year-olds were still problem areas for the 17-year-olds. (p. 34)

The difficulty in learning decimal number concepts may be owing to: a lack of firm understanding of the place-value interpretation of decimals (Lichtenberg & Lichtenberg, 1982; Nichols & Behr, 1982), no semantic meanings for the symbol (Hiebert, 1984; Hiebert & Wearne, 1987; Hiebert & Wearne, 1984; Lesh & Landau, 1984), or the fact that school programs tend to emphasize procedural skills and computational aspects rather than the development of important foundational understandings (Post, Behr & Lesh, 1982). A study in Israel (Tomer, Wolf, Ron Hoz, 1987) with forty-seven sixth to ninth graders of average mathematical ability, attempted to develop the conceptual understanding of the relationships between the systems of common fractions and decimal rationals. It also studied the procedures for the transition between the two systems. The results showed that most deficiencies and misconceptions persisted from grades six to nine.

The work of Hiebert & Wearne has been the most comprehensive attempt made to delineate the cognitive aspects of decimal number knowledge. Hiebert (1984) points out that many children do not connect the mathematical concepts and skills which they know intuitively with the rules of mathematics that they learn in school. He distinguishes two types of knowledge that children acquire about mathematics: form and understanding. Form includes symbols for numbers, operations and relations. Understanding involves intuitions and ideas that make sense to children about how mathematics works. Some of these are learned in school, and some arise from everyday situations.

Hiebert (1984); Hiebert & Wearne (1986) describe three levels or sites in the problem solving process where form and understanding might be linked. Site 1 is the initial point in the problem-solving process, when the problem is interpreted. It is the point at which the symbols of the problem are given some meaning, therefore at site 1, the symbols can be linked to referents that give them meaning. Once the problem has been interpreted, procedures can be selected and applied in order to solve that problem. The execution of the procedures happens at site 2. It is at site 2 that form and understanding are linked so that students can connect a procedure with the underlying concept or rationale that motivates the procedure. Once the procedures have been executed, a response is produced that represents the student's answer to the problem. At site 3, the written solution to the problem should be checked against other knowledge students have to provide some clues about the reasonableness of the solution.

Hiebert (1984) has interpreted the difficulties that students exhibit at these three sites as being a divorce between procedural and conceptual knowledge. It should be noted that many categories have been used to define understanding; however, this work will use the terms procedural and conceptual knowledge. Students memorize rules for manipulating symbols that aid in decimal computation. Students learn important concepts about the decimal system and also bring with them intuitions about mathematics that they have acquired in and out of school. However, students, in most cases, fail to see any connection between the mathematics they have learned in school and their intuitions. No links have been established between these two domains of knowledge.

Many learning problems in mathematics can be attributed to the absence of connections between the memorized, mechanically applied rules and conceptual understandings (Hiebert & Wearne, 1984). How can these connections or links be attained? The critical instructional problem may be one of helping students connect pieces of information that they already possess.

Purpose of the Study

This study sought to define the kinds of techniques that young adults (Math 15 students) use when involved in solving decimal computation problems and then provide experiences which allowed such students to reconstruct decimal knowledge.

By nature, the Math 15 curriculum reviews topics that have been studied in junior high school. The Math 15 curriculum appeals to a particular group of high school students. These students are usually low on motivation, self-esteem and confidence. They have a history of poor performance in mathematics. Math 15 is the minimal mathematics course available to students who wish to obtain a high school math credit in order to fulfill the requirements of the Province of Alberta. The design of the curriculum is for this course to give students help with previously poorly learned material and to provide an introduction to advanced material such as trigonometry. Math 15 students have been chosen for this study because by nature of the curriculum, they are applying previously taught skills to relearn and reconstruct their mathematical knowledge.

Math educators know that Math 15 students have difficulty with mathematics. The history of these students shows that many of them perform poorly on various mathematical tasks. This study attempted to find the interrelationship between the structure that the students have and the outcome of instruction on this structure. The study also attempted to describe this relationship theoretically. Two means were used to accomplish this:

1) to find out what students' held knowledge was, and

2) to design a situation where these students would come face to face with their misconceptions and observe what these students did under these circumstances.

This research attempted to look at the reconstruction of decimal knowledge under particular conditions with a specific group of students. The group of students had attempted to understand the decimal number system before and had little success. It was hoped that through this study, it would be possible to build onto the existing decimal knowledge held by the students. This study examined the learning process in terms of a proposed learning model and it was hoped that this model could be applied to the teaching process as well.

Nature of the Study and Identification of the Researcher

The nature of the study was to observe the growth and development of these students' mathematical work with decimals. This study presented a detailed observation of students attempting to know and understand under particular learning conditions. This work fits the researcher's interest both as a teacher and as a researcher. As a teacher, comments were often made concerning the difficulties students seemed to be experiencing when working with decimals. As a researcher, the question was studied to try to discover what the difficulties were, why the difficulties existed and what could be done about the difficulties.

Definition of Terms

The following definitions pertain to the original work in the study. If these terms are used any other way it is because they are taken directly from the work of other people.

Conceptual knowledge - knowledge of those facts and properties of mathematics that are recognized as being related in some way.

Decimal fraction - a fraction in which the denominator (not written) is a power of ten.

Decimal number - equivalent to decimal fraction.

Error - the student committing an error misuses a procedure or makes a wrong record. The student can immediately recognize this result (at least when shown) and can correct it.

Fraction - the symbol a/b where a and b are natural numbers and $b \neq 0$

Fractional number - an equivalence class of fractions.

Math 15 student - a student enrolled in the Math 15 high school program. A detailed description is found on page 3.

Misconception - the student acts in a systematic way which leads to incorrect results in the eyes of the observer.

Procedural knowledge - knowledge of how written mathematical symbols behave according to syntactic rules.

Reconstruction - the ability to encounter and recognize an existing misconception and make some changes to this schema in order to accommodate new information that will lead to a correct application of the new information.

Teaching experiment - a period of closely monitored teacher directed activities designed to allow the teacher/researcher to observe student thinking under instruction. The activities were designed with the student misconceptions, observed on the pretest and the preliminary interviews, in mind. The intention of the activities was to help students discover misconceptions and change their understanding. The teaching experiment is elaborated upon in Chapter V.

Delimitations

This research examined the reconstruction of decimal knowledge under particular conditions. The following delimitations were imposed upon the study:

1. The study is delimited to Math 15 students.

2. There are many aspects that pertain to decimal knowledge; however, this study dealt with only a selected few of these aspects. They include:
   - place value
   - estimation
   - operations
   - scale reading
   - comparing decimals
   - fraction-to-decimal translation.

These topics have been selected because they are the aspects of the decimal number system that the students have previously studied and used most in their application of their decimal learning.

Limitations

There were four major limitations to this study. The "think aloud" method of determining processes utilized by the subjects in solving problems might have been unreliable since an individual could remain silent during moments of deepest thought. A verbalized solution could be different from one effected silently.

The presence of the researcher could have put constraints on a student in that he/she might not have attempted solutions for fear of feeling foolish or insecure.

At the beginning of the study, the students wanted to remain anonymous and therefore, did not put their name on the pretest. The pretest results of the entire Math 15 class were used to study growth that occurred during the study even though the exact pretest results of the eight students participating in the study were not known. Therefore, the pretest mean of the twenty-nine Math 15 students is used throughout the study even though the study involved only eight students. It should be noted that this information was used for situational purposes and not for statistical conclusions.

During the pretest interviews which took place shortly after the written pretest, a few of the students who were interviewed preferred not to the interviewed alone. So as not to put pressure on the students the researcher agreed. Therefore the pretest interviews were not all compiled in the same way. This limitation is discussed on page 31.

The study was limited to four weeks of instruction which took the form of worksheets. The justification for this limitation can be found on page 26.

Design of the Study

The following is a brief description of the study. A detailed description is presented in Chapter III.

Ascertaining the decimal number knowledge of Math 15 students occurred in two stages. A paper-and-pencil test was administered to a class of students enrolled in Math 15 in the City of Edmonton. A subsample of these students was selected to be interviewed. The interviews permitted the students to use a "work-aloud" strategy to solve problems similar to items on the written test. The interview sessions provided information concerning the ways in which the student viewed the problem, and the procedures that were used in attempting to solve the problem.

On the basis of the test responses and the interview data, a teaching experiment was designed to help the students "construct the kind of knowledge framework which is necessary to correct handling of the problems in question" (Kerslake, 1986, p. 6). A post-test was administered to these students immediately after the teaching experiment, followed by a delayed post-test ten weeks later to determine the results of the teaching experiment.

Significance of the Study

The study has sought to gain insights into the kind of decimal knowledge held by a particular group of students. It is hoped that these insights, combined with the results of a teaching experiment that focuses on the reconstruction of decimal knowledge in these students, will contribute to the knowledge and understanding of decimal knowing and will provide a model that may help educators understand why students in Math 15 have difficulty learning mathematics.

It is hoped that this model can help educators determine how these students can rebuild their decimal knowledge. There exist different theories that deal with the various stages of development. One of the best known theories is that of Jean Piaget. However, it is felt that these theories do not handle the growth of these particular students. It is hoped that this theory will help bridge that gap.

Organization of the Dissertation

The present chapter is an outline and preview of the study. A review of the relevant literature is presented in Chapter II. It includes a section concerning the cognitive processes of decimal number learning, and an examination of the curriculum and instruction of decimal numbers. Also included in Chapter II is a section that examines a model of decimal number learning labelled by the researcher as the Onion

Model of Decimal Number Learning.

Chapter III contains a description of the research design, the data collection and analysis process. The findings of the study are presented in Chapter IV and Chapter V analyzes the teaching experiment. The students' worksheets are analyzed using elements of the Onion Model in Chapter VI. The concluding chapter, Chapter VII, includes a summary, discussion of the results, reflections on the teaching experiment and reflections on the Onion Model.

CHAPTER II

Review of the Literature

Introduction

The purpose of this research was to ascertain the decimal number knowledge of Math 15 students, to study students' knowledge as it changed during a teaching experiment, to identify inter-relationships between held knowledge that the students possessed and knowledge growth under instruction, and to try to characterize such knowledge processes and inter-relationships theoretically. The purpose of this chapter was to develop a theoretical framework for the study based on research in the field.

Research on the topic of decimal number knowledge has focused mostly on procedural knowledge and computational skills. It has included large-scale national surveys, one in the United States (Carpenter, Corbitt, Kepner, Lindquist, & Reys, 1981) and one in England (Brown, 1981; Hart, 1981); results of instructional lessons designed to remediate misconceptions (Bell, Swan, & Taylor, 1981); individual performance on decimal tasks (Lichtenberg & Lichtenberg, 1982; Zawojewski, 1983; Grossnickle & Perry, 1985); and approaches to the teaching of place value and decimals (Swan, 1983). A major source of children's understanding of decimal number knowledge is the work done by Hiebert and Wearne which is discussed in some detail in this chapter.

The chapter reviewed two major areas of research: research on the cognitive processes of decimal number learning, and research on curriculum and instruction of decimals. Finally, a section was included which summarized the theoretical framework as it emerged from the study.

Cognitive Processes of Decimal Number Learning

This section summarized the research into the cognitive aspects of decimal number learning.

The work of Hiebert and Wearne has been the most comprehensive attempt made to delineate the cognitive aspects of decimal number knowledge. Hiebert & Wearne (1986) argue that:

> Mathematical competence is characterized by connections between conceptual and procedural knowledge ... that mathematical incompetence often is due to an absence of connections between conceptual and procedural knowledge. (p. 199)

Questions of how students learn mathematics, and how they should be taught bring about speculations about which type of knowledge (conceptual or procedural) is more important or what might be an appropriate balance between them. Cognitive psychologists and mathematics educators are examining conceptual and procedural knowledge in mathematics learning (Hiebert & Lefevre, 1986).

Skemp (1976) speaks of "relational understanding" and "instrumental understanding". Relational understanding is knowing both what to do and why; instrumental understanding is described as rules without reason. Whether conceptual/procedural knowledge or instrumental/relational understanding are spoken of, it is not certain how to establish links between the two. Skemp (1976) poses two questions which are considered to be very important: Does it matter? Is one kind better than the other?

Conceptual knowledge is knowledge of those facts and properties of mathematics that are recognized as being related in some way by the person using those facts and properties. When a fact or property becomes part of a larger network through the recognition or construction of a relationship between that fact and a network that is already in place, then it can be said that that fact becomes part of conceptual knowledge.

For example, students are taught the names of the position values of whole numbers (ones, tens, hundreds, etc.). If this knowledge is acquired and stored as individual bits of information that serve only as labels for the value of each position, it does not qualify as conceptual knowledge. However, if this knowledge of place value becomes linked to other mathematical knowledge already known, such as counting by tens, and hundreds and grouping objects by tens, then it becomes conceptual knowledge.

Procedural knowledge is limited to knowledge of how written mathematical symbols behave according to syntactic rules. Procedural knowledge of symbols does not include knowing what the symbol means; that is, knowing that the symbol represents an external referent. Procedural knowledge also includes the set of rules or algorithms that are used to manipulate the symbols and solve mathematical problems.

However, because mathematical ideas are conveyed using specialized, compact symbol-systems it is advantageous to be able to manipulate these symbols. Skemp (1989) does not feel that anything is learned simply from manipulating symbols. Manipulating symbols contains meaning only when the conceptual structure is understood:

> Symbolic understanding is a mutual assimilation between a symbol-system and an appropriate conceptual structure. (p. 98)

If the conceptual structure is weak or barely existent, then the symbol-system dominates and mathematics becomes the manipulation of symbols.

Pimm (1987) notes the importance of being able to distinguish between the symbol and the concept. Being able to make this distinction allows the manipulation to move faster and be more effective. Both the symbol and the concept have their special attributes.

A study conducted by Resnick et al. (1989) documents major categories of errors that appear consistently as children learn decimal fractions. They have investigated whether children's efforts to make conceptual sense of new mathematics instruction in terms of their already available knowledge may sometimes lead them to make systematic errors. They explore the possibility that children overgeneralize concepts from a familiar domain of mathematics (e.g., whole numbers) in order to interpret a new domain (e.g., decimals). Resnick et al hypothesized that children's errors may derive from their efforts to apply previously learned concepts or notational conventions to a new domain. Children may feel that because the new decimal concepts share key features of place value with whole numbers, that the two systems are identical and lead them to ignore the differences between the two systems. This feeling that the two systems are identical may be heightened by teachers' attempts to help children use prior knowledge of whole numbers to facilitate learning of the new decimal system whereas knowledge should be built through distinctions. Children who have learned ordinary fraction notation before decimal numbers may attempt to apply their conceptual knowledge about the relationship between size of parts and number of parts to an incomplete understanding of the referents of decimal notation. Because thousandths are smaller than hundredths, a child may infer that any 3-digit decimal number will be smaller than any 2-digit decimal number. Once the child's rules have been established, the research team tries to uncover the child's conceptual frameworks, with particular attention to whether they are thinking of decimals in whole number or ordinary fraction terms.

Their (Resnick et al., 1989) conclusion shows that children are often taught syntactic rules which serve to suppress errors in performance without improving children's conceptual understanding. Several analyses have shown that these erroneous rules are intelligent constructions based upon what is more often incomplete than incorrect knowledge.

In a study to investigate middle school students' thinking about decimal fractions (Wearne & Hiebert, 1987), the most significant finding is that most students recognize few, if any, connections between their conceptual knowledge of decimals and the symbol-manipulation rules they have used to solve problems. The absence of links between concepts and procedures is so extensive and pervasive that the two kinds of knowledge can be characterized as making up two separate mental worlds.

Hiebert & Wearne (1985) hypothesize that students compute by relying solely on syntax-based rules and that semantic knowledge has no effect on performance. As previously mentioned, many children perform mathematical calculations by applying memorized rules to manipulate symbols. Apparently, many students also continue to operate almost entirely from a procedural level through high school and beyond. A model "that describes the construction and execution of decimal computation procedures" (Hiebert & Wearne, 1985, p. 176) is presented. The model is made up of paper-and-pencil symbol manipulation rules and "describes computation as a process in which individual rules are selected and executed in sequence" (Hiebert & Wearne, 1985, p. 177). The individual rules referred to in the model are those taught in school for working with decimal numbers. These rules are used to manipulate symbols. If students have been taught the rules, then, with each calculation, the student must select a rule to use. Hiebert & Wearne propose three sources that activate rules. One source of activation is the task goal. If the student is faced with a division problem, the rules he/she has learned for division will be primed. A second source of activation is the amount of practice that the student has had using a rule, and the third source of activation is learning recency. A rule that has been recently learned and practiced is still fresh in an individual's mind. Hiebert & Wearne (1985) have hypothesized that students derive wrong answers because they select inappropriate rules rather than because they execute the rules incorrectly. Hiebert & Wearne's conclusion is that students' procedures are indeed syntactically based:

> It appears that students' improved performance with age results from an increasing facility with the syntax rather than from a better conceptual understanding of the procedures. (Hiebert & Wearne, 1985, p. 200)

The generative theory of Brown & Van Lehn (1982) suggests that students construct erroneous rules without reference to the conceptual content. Brown & Burton (1978) state that "students are remarkably competent procedure followers, but that they often follow the wrong procedures" (p. 157). Brown & Van Lehn's (1982) generative theory demonstrates that many bugs (erroneous variations of a procedure) can best be explained as being "patches derived from repairing a procedure that has encountered an impasse" (p. 123). This theory is motivated by the belief that when students attempt a procedure unsuccessfully, they will undertake to repair or fix that procedure. If students are missing a part of the correct procedural skill and that missing part has a purpose, then the students will arrive at an impasse. When the learners arrive at an impasse, they will either give up or, in most cases, will attempt to overcome the impasse and continue with the procedure, albeit in an erroneous way.

Hiebert & Wearne (1984) indicate that there are three levels, points or "sites" in the process of computing with decimal numbers that demarcate the primary sources of students' difficulty. At site 1 many students do not know what the symbols mean. They fail to connect decimal symbols with meaningful referents. A study by Hiebert (1985) demonstrates that most fifth and sixth graders are unable to extend visual models of whole numbers (pictures of bundled popsicle sticks) to create referents for decimal symbols. At site 2 many students do not know why the computation procedure works. Based on individual interviews and analysis of written errors (Hiebert, 1985), most students' computation activity consists of recalling and applying memorized rules to which they connect absolutely no rationale. Perhaps students' background in mathematics and their cognitive capabilities are not sufficiently developed for them to understand the complexities of these algorithmic procedures (Hiebert & Wearne, 1986). At site 3, many students are not aware that answers should be reasonable. To be able to check whether an answer to a decimal computation problem is reasonable, an individual must connect at least an intuitive idea of the arithmetic operation with appropriate meanings for symbols. Hiebert & Wearne (1987) interpret the difficulties that students exhibit at each of the three sites in the computation process as a consequence of a divorce between procedural and conceptual knowledge.

Hiebert & Wearne (1987) have tested a carefully developed theory of symbolic knowledge building. They have proposed that competence with written mathematical symbols develops through the sequential and cumulative mastery of four distinct cognitive processes. The first two processes develop the semantics of the particular symbol-system and the second two elaborate its syntax. The four processes include: (1) creating meaning for individual symbols by connecting them with familiar or meaningful referents; (2) developing symbol manipulation procedures by reflecting on the meanings of the symbols; (3) elaborating and routinizing the procedures and rules for symbols; and (4) using the symbols and rules as referents for building a more abstract symbol system. Their objective is:

> ... to provide explicit opportunities for the student to acquire processes that create meaning for written symbols, and processes that use these meanings to guide the development of simple procedures on symbols. (p. 392)

Two studies have been conducted. Students in the first study are instructed in small groups while students in the second study are instructed as an entire class. The results of this study show that students use appropriate semantic processes after instruction but in the full classroom group, the level of transfer to related decimal tasks before direct instruction is low. That is, most of the students do not extend or transfer the semantic processes that they have discussed early in the instruction to tasks that appear in later activities.

Hart (1981) has conducted an extensive study to develop a hierarchy of understanding in mathematics. Eleven mathematical topics are tested and evaluated and from the written test results, a set of items (within each topic) has been identified which appears to form a relatively homogenous group. These are then divided into six levels. A student is said to have attained that level if he or she has answered two-thirds of the items at that level correctly. The aim of the test on Place Value and Decimals is to find whether children can meaningfully use the base-ten place-value notation for both whole numbers and decimals, in the sense of understanding how it works and applying it to appropriate situations. From the results, it can be generalized that by the time students leave school the top fifty percent of pupils are likely to have a reasonable if not complete understanding of decimals, whereas the lower fifty percent of the pupils still have considerable gaps in their understanding.

Curriculum and Instruction of Decimal Numbers

According to the National Assessment of Educational Progress (NAEP) (Carpenter et al., 1981) students perform poorly on tasks involving decimal computation. Most nine to thirteen-year-olds lack a firm understanding of the place-value interpretation of decimals; they have not firmly established a relationship between decimals and common fractions; and lack conceptual understanding of decimal number knowledge which affects performance with decimal computation and application. Results of the NAEP regarding decimals indicate that students have not mastered the concept of decimal number knowledge by age thirteen and that many of the areas of difficulty for thirteen-year-olds are still problem areas for seventeen-year-olds.

Research based on the instruction of decimal numbers is fairly recent. The studies reviewed here incorporate decimal misconceptions with instruction on decimals. Lichtenberg & Lichtenberg (1982) report that:

> The typical approach to decimals does not allow enough time for developing meaning, whereas inordinate amounts of time are devoted to the computational procedures. (p. 143)

Here the emphasis is on teaching computational skills and on how to manipulate the symbols to arrive at a correct answer.

Lichtenberg & Lichtenberg have used the monetary system, square regions, a chain made from 100 colored plastic links and the abacus as models for representing decimals. These models have been used in an attempt to help students correct certain errors such as replying that .09 > .1. It is suggested that although computational processes are easier to teach, teachers must find time to spend on developing the concepts underlying decimal number knowledge.

Zawojewski (1983), in a study designed to look at the initial learning of decimal concepts at grades four, five and six, have found that:

1. emphasis should be placed on models and oral names for decimal numbers,

2. the initial learning of tenths and hundredths should be separated from the work with place-value, and,

3. there may be a factor of readiness involved.

The results of the above studies seem to indicate that decimal number knowledge is being taught by means of computational processes and that most students memorize an abundance of task-specific rules for manipulating symbols for which they have not acquired meanings. Hiebert (1984) sees the critical instructional problem not as one of teaching additional information, but rather as one of helping students see connections between pieces of information that they already possess.

A comparative experiment of two different teaching styles in the curriculum of decimal place value was conducted by Swan (1983). The "conflict" teaching approach encourages students to make, discuss and reflect on their own misconceptions and errors, whereas the "positive only" approach attempts to avoid any errors occurring by providing simple and understandable methods from the start. Results of the study have shown that:

1. both programmes are effective at enabling students to understand decimal place-value, and,

2. the "conflict" programme is significantly more effective at permanently removing and correcting misconceptions.

Bell & Purdy (1986) have conducted interviews and tests with students looking carefully at their mistakes and using these to help them understand the ways of thinking that lie behind students' errors. The chief misconceptions identified in this study (Bell, 1986) are the failure to accept the possibility of dividing a smaller by a larger number, and the assumption that multiplication makes bigger and division smaller. To help students overcome these misconceptions teaching materials have been prepared which aim at three aspects: notational misconceptions, numerical misconceptions and the invariance of quantity relations in problems in contexts such as price and speed. The assumption is that if the difficulties with notation and the numerical misconceptions can be overcome, then correct choices of operation will be made. However, in the two series of experiments conducted this was not the case. Improvement takes place on numerical questions (e.g., is 19.4 x 0.65 greater or less than 19.47) but not on choice of operation in problems. Direct attention to problems in context, such as price and speed, are needed.

Kieren (1987), in his discussion of fraction number research papers, comments that several of the studies have shown that traditional instruction makes an early and probably unwarranted emphasis on symbolic manipulation and computation with common or decimal fractions. If this is true, then children are probably forced to treat these symbols as concrete objects and thus build knowledge based inappropriately on patterns in the symbols (e.g., count the number of decimal places). What seems clear is that a fraction curriculum in which symbols are not tied to meaningful object actions has inhibiting effects.

Owens & Haggerty (1987) have observed the processes of children as they form concepts and attach meaning to the multiplication of decimal fractions. Children are often taught to count the places after the decimal point in order to place the decimal in the product. Such algorithmic strategies can be used without understanding and their use may lead to difficulties. An examination of the processes used by the students reveals that a variety of strategies is used to multiply decimal numbers. Each strategy represents a particular construction of the concepts at hand. In many instances, background concepts have not been mastered and the resulting constructions do not always lead students to correct answers. In some cases students have confused addition/subtraction strategies with multiplication strategies when they are placing decimal points. Owens' report raises a pertinent issue. It is known that children can and do learn rules for computation. This study raises serious questions about the quality of understanding that can be expected under normal classroom conditions for children in grade five.

Perhaps the quality of understanding that can be expected in Math 15 students if they are being taught by means of computational processes should be questioned. It is hoped that this study will provide some insight into the decimal number understanding that can be attained by these students and by other students.

Summary

The literature reveals that there are two distinct sets of knowledge - conceptual knowledge and procedural knowledge. Of these two sets of knowledge, procedural knowledge is the more prominent. One reason for this seems to be that most decimal number knowledge is taught by means of computational processes dealing with the manipulation of symbols leaving the students no choice but to memorize an abundance of procedures that have no conceptual meaning.

The data from a Decimals Diagnostic Test [used with permission from Swan, (1988)], reveals that procedural knowledge dominates the students' decimal number knowledge. Therefore, there is need to examine what procedures the students know, how they are using them and whether it is possible to arrive at conceptual understanding. The means by which students can be assisted to arrive at conceptual understanding has been sought by way of a teaching experiment consisting of teacher directed activities that allow the students to solve and discuss problems concerning aspects of decimal number knowledge.

Theoretical Conclusion of the Literature

The following section has evolved as a result of the literature review and data collected from two pilot studies. A pre-pilot study has been conducted in March, 1988 and a pilot study has also been conducted in May, 1988. Each study has been carried out in a Math 15 classroom in a different high school in the City of Edmonton. Both studies involve a pretest, pretest interviews, worksheets dealing with the topics on the pretest and a post-test. It has been noticed in both of these studies that a period of teacher-directed activities leads to the correction of certain decimal knowledge misconceptions. The background for this model comes from readings, preliminary data collection and observations in a Math 15 classroom. It is important to be able to analyze students' knowing and understanding. One way of doing this is by looking at change over time, a growth model. The Onion Model is one interpretation of how decimal number knowledge and understanding "grows". As noted on pages 17 and 18, one such type of growth model is a layered model. This layered model looks at change between layers and change within layers.

This model of Decimal Number Learning supposes that conceptual knowledge can be attained. However, in order to reach conceptual knowledge of decimal numbers an individual must pass through various layers. The purposes of this model are to define these layers, ask questions about these layers and relate them to the curriculum and instruction of decimal number knowledge.

ONION MODEL ON DECIMAL NUMBER LEARNING

With the examination of various research studies on decimal number knowledge, decimal number learning and with some preliminary data collection, evidence can be found that for a majority of students, decimal knowledge is superficial. Students can read a decimal number and can correctly tell the place value name for each position in a decimal number. Most students are capable of carrying out decimal number computations by applying learned rules and the pupils function quite well in the area of decimal computation. However, there seems to be no depth to what the students are doing and they do not understand why they are doing it. For example, they fail to see the relationship between the place values of a decimal number; they sometimes ignore the decimal point and treat decimal numbers as whole numbers; they are not aware of what zero as a place holder means; and they are confused about the "ten-ness" of a decimal.

Research studies are concerned with the acquisition of conceptual understanding of decimal numbers. These studies indicate that procedural knowledge is more prevalent than conceptual knowledge and that students have difficulty going beyond procedural knowledge. The findings that have been taken from these studies are that mathematical knowledge and understanding can be defined through the use of levels. Hiebert & Wearne (1984) speak of levels, points or sites that demarcate the primary sources of students' difficulty and of four processes that help to develop competence with written mathematical symbols (1987). Hart (1981) has divided a range of questions about mathematical topics into six levels.

The foregoing research has focused on these aspects: layers of development, learning hierarchies, and procedural knowledge versus conceptual understanding. There is an indication that students move through some kind of sequence but that the understanding is not seen as a whole. The researcher has tried to develop a model for understanding as a whole where the whole represents a sense of complete understanding. To do this, the metaphor of an onion has been used.

The Onion Model had three purposes:

1. it served to tie together the research,

2. it allowed the researcher the opportunity to observe how the students were thinking (both before and after the teaching experiment), and

3. it allowed the researcher to bring together some of the results.

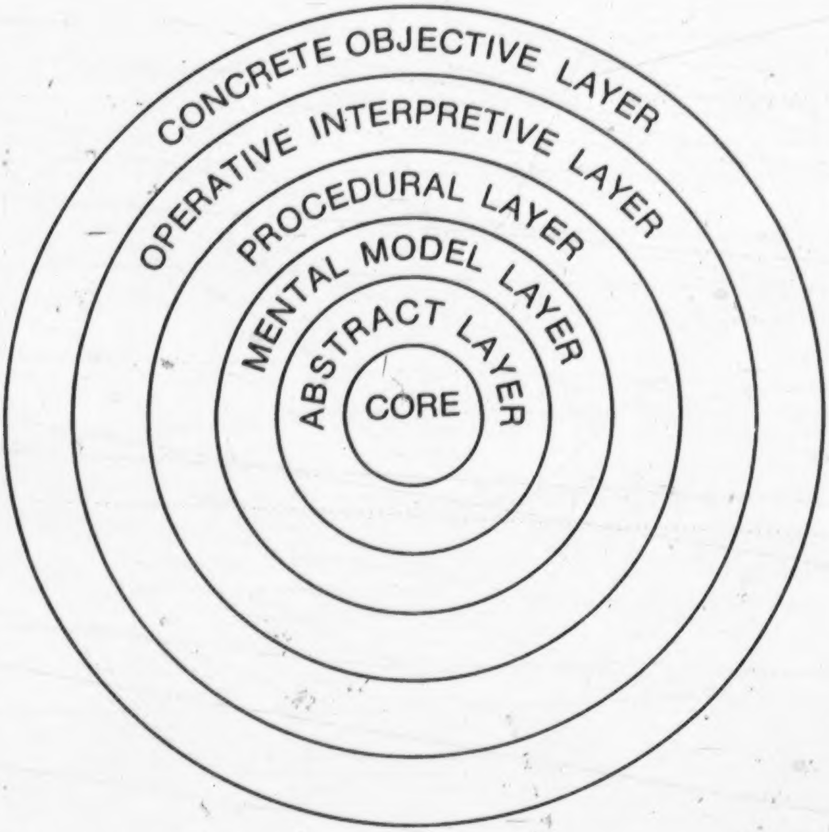
The onion model of decimal number learning incorporates features of the research and suggests that conceptual understanding is at the very core of decimal knowledge. Arriving at the core is akin to peeling an onion. The core is not a layer in itself. It signifies that once a person has peeled the abstract layer, he/she has arrived at the core which represents complete understanding. The core is surrounded by layers that must be peeled before the core can be reached. The layers represent different kinds of understanding acts. The peeling of a layer represents the fact that a person has accomplished the kind of thinking necessary for that particular layer. Once a person has peeled all the kinds of understanding acts, he/she has reached complete understanding, metaphorically represented by the core. One or more than one layer at a time can be peeled but if a single layer cannot be peeled, then it will be very difficult to reach the core of decimal knowledge (conceptual understanding).

Before the core can be reached there are five layers to be peeled (Figure 2:1). Some of these layers may be easier to peel than others and with certain students it may be impossible to peel a certain layer.

The first layer is the concrete-objective layer. The concrete-objective layer represents the stage at which manipulatives, visual aids and real world situations are employed to enter the world of decimals; it involves knowing the nature of the objects. As an example Dienes' multibase blocks can be used to illustrate decimals in a place value setting. If the flat were defined as the unit, longs would be tenths and units would be hundredths. If the block were defined as the unit, flats would be tenths, longs would be hundredths, and units would be thousandths. An important number-related skill is that students can easily redefine the unit.

The second layer, the operative-interpretive layer, represents the point at which the teacher moves from the concrete level to the instructional level using symbolic

Figure 2:1
ONION MODEL OF DECIMAL NUMBER LEARNING


CONCRETE OBJECTIVE LAYER
OPERATIVE INTERPRETIVE LAYER
PROCEDURAL LAYER
MENTAL MODEL LAYER
ABSTRACT LAYER
CORE

representations of decimal numbers. This layer includes the teaching of what a decimal is and how decimals are worked with. The layer involves interpreting things that people can do with the objects involved with the first layer. For example, the multibase blocks can be used for addition with the flat defined as the unit. The sum of 0.6 and 0.73 would be one flat, three longs and three units after exchanging ten longs for one flat.

The third layer, the procedural layer, is the point at which the students use the instruction they have received at the operative-interpretive level to compute with decimal numbers. This layer introduces procedures that can be applied when working with decimals (e.g. counting places when multiplying two decimals). The procedural layer allows students to perform operations using symbols alone.

There seems to be a layer between the procedural and the abstract layers. Most students can perform quite well at the procedural layer but are unable to reach the abstract layer. Being able to use a procedure is one thing; knowing how and why people do it is another. At the procedural layer students can follow a set of rules but they do not look back to see what they have done nor do they verify what they have done. The mental model layer is the stage where students take the understanding that they have to do a sequence of individual tasks and can realize what they have done. Students are able to look back and understand what has been done. At this stage students have an independent mental model of what is going on and, in order to function independently, these individuals would have to have some idea of what is going on besides just being able to follow directions.

The fifth layer, the abstract layer, is the stage at which students no longer need visual aids; they have an intuitive sense of what decimals are and can estimate correctly when doing decimal computations. The "how" and the "why" are integrated. Once this layer has been reached it can be peeled and the students have attained the core of decimal knowledge, conceptual understanding.

Abstract understanding is a combination of the form and understanding of decimals. Students at the procedural layer can use a set of facts only in the circumstances in which they belong; whereas, students at the abstract layer can explain and predict a greater number of events under a greater number of circumstances. They have the ability to reason about the hypothetical and the real and have the capability to deduce conclusions from hypothetical premises. Students who have reached this layer can take completely new situations that involve decimals and figure them out. Students at the abstract level could, for example, tell that 1/10000 x 1/100 is the same as 1/1000 x 1/1000 without computing the answer. However, students at the procedural level will solve the question with a calculator and will have no other means of checking the answer. It is at the abstract layer that students have taken all of their thinking, formalized it and replaced it with an algorithm. These students have now found other ways of solving the question without doing all the thinking. The abstract layer involves not only having a conceptual model but realizing that people have their own conceptual models; these are people's ideas of what decimals are and how they can be used.

Perhaps something happens within the first three layers that serves as a blockade to the abstract level. How are teachers treating the first three layers? Are they being treated properly? These concerns lead to questions concerning the concrete-objective, operative-interpretive and procedural layers.

At the concrete-objective layer the following questions can be asked:

- Should more time be spent at the concrete-objective layer?
- What is being done at the concrete-objective layer?
- What should be done at the concrete-objective layer?
- How are concrete materials being used at the concrete-objective layer?
- Can concrete materials be used more effectively at the concrete-objective layer?

At the operative-interpretive layer the following questions can be asked:

- What decimal knowledge is being taught?
- What instructional techniques are being used?
- What is being accomplished by the various instructional techniques?
- Is one instructional technique more effective than another for the teaching of decimal computation procedures?
- How can instructional techniques be improved to go beyond being able to compute mechanically with decimal numbers?

The procedural layer is closely related to the operative-interpretive layer as it is at this layer that teachers tell students what operations will be done at the procedural layer. If improvement can be made at the operative-interpretive layer then improvements at the procedural layer are certain to occur. One critique of the curriculum is that it starts a person at the procedural level. It ignores what decimals are and begins with teacher "doing" because "doing" decimals is fairly easy to accomplish. If people start at this level they appear to be beginning in the dark. If it is impossible to pass through the first two layers and start at the procedural layer, what are the chances of going further? What would it take to get to the abstract layer? The question can also be asked as to whether these layers exist without each other. The proposed model offers a research-backed conception of how students acquire and develop decimal number knowledge. The various layers contained within the model suggest that a student passes through these layers in order to attain conceptual understanding of decimal numbers. The model also suggests that, for some students, something happens during the first three layers that does not allow them to go on and reach the abstract layer which will lead them to the core or conceptual understanding. The responses to the teaching experiment will act as a guide to help place each student in the study at a point represented by the model.

Some students do not seem to be able to go beyond the procedural level. It must be determined whether the problem is mental or instructional. This model implies that students cannot reach the abstract layer before the mental model layer is peeled away and before the mental model layer can be peeled away it is necessary to find out just what it is that makes this layer so difficult to peel.

CHAPTER III

Methodology and Procedures

Introduction

The purposes of this study were to 1) ascertain decimal knowledge of students enrolled in Math 15, 2) study students' knowledge as it changed during a teaching experiment, 3) identify inter-relationships between held knowledge that the students possess and knowledge growth under instruction, and 4) attempt to characterize such knowledge process and inter-relationships theoretically. The review of the literature revealed that students performed poorly on tasks that required decimal computation (Carpenter et al., 1981) and that students' decimal number knowledge consisted of an abundance of memorized procedural skills. An extensive study (Hart, 1981) conducted to develop a hierarchy of understanding in mathematics resulted in the generalization that the top fifty percent of pupils are likely by the time they leave school to have a reasonable if not complete understanding of decimals, whereas the lower fifty percent of pupils still have considerable gaps in their understanding. This study attempted to find out why such gaps existed and how the gaps might be bridged. This chapter includes a description of the sampling process and a discussion of the research procedures employed for the study.

The Subjects

The study was conducted during the months of November and December, 1988 and took place in a public high school in Edmonton, Alberta. All subjects were enrolled in Math 15. A pretest was administered to the entire group of Math 15 students (n=29) and eight subjects were chosen on a volunteer basis from this group to participate in the teaching experiment. The eight subjects were separated from the rest of the class and went with the researcher into another classroom. The remainder of the class remained with the regular teacher and worked through the same activities as the subjects. It is not known how the regular teacher used the activities. This study interpreted the interactions and growth of the eight students who took part in the teaching experiment.

The Procedure

The study was comprised of two phases. The first phase involved a pretest as well as pretest student interviews to ascertain the decimal number knowledge of the students. The second phase involved the teaching experiment, the post-test, the delayed post-test and the delayed post-test student interviews for the purpose of looking at the knowledge and for observing any changes in the knowledge structure of the students. Each phase is described in more detail in the following section.

The Pretest

One purpose of the study was to ascertain the decimal number knowledge of students enrolled in Math 15. Ascertaining the decimal knowledge of these students occurred in two stages - a pretest and interviews. The pretest (Appendix A), modified from Malcolm Swan's Decimals Diagnostic Test (Swan, 1983, 1988) was used to ascertain the decimal knowledge of the subjects. The entire Math 15 class (n=29) was given a written test on chosen aspects of decimal knowledge. The written test included items on decimal computation and a variety of items on the different meanings that could be attributed to decimal numbers such as: relationship to fractions, place value and measurement. The items were designed to elicit student's knowledge of the topic. The pretest served to identify a base level of decimal number knowledge and application of the decimal concepts tested. The pretest was administered in an uncompetitive atmosphere and the students were informed that the test was not to become part of their grade mark.

Interviews

A sub-sample of the students who wrote the pretest were interviewed in order to ascertain their conceptual knowledge of decimals. Nine students volunteered to be interviewed and eight of those volunteered to take part in the study. The interview sessions provided information concerning the ways in which the students viewed the problem, and the procedures and processes that were used in attempting to solve the problem. This information was of interest because it sought ways of helping students avoid these errors and helped to explain students' lack of progress in attaining higher levels of decimal number knowledge.

The interview involved interaction with individual students as they worked their way through problems that appeared on the pretest. This allowed the researcher to find out what processes the student had used on the pretest. A "work-aloud" strategy enabled the researcher to view the process used by students when solving particular problems. The interactive encounters were designed to yield insights in the way students thought of decimals. The interaction was characterized by flexibility in questioning. This permitted probing, following through students' behaviors, expanding on a question or altering the problem.

The interviews were audio recorded and analyzed based on a method suggested by Glaser and Strauss (1967):

> Theoretical sampling is the process of data collection for generating theory whereby the analyst jointly collects, codes, and analyzes his data and decides what data to collect next and where to find them, in order to develop his theory as it emerges. This process of data collection is controlled by the emerging theory, whether substantive or formal. (p. 45)

A summary of the interviews appears in Chapter IV and in Appendix B.

The Teaching Experiment

Having identified aspects of decimal knowledge with which pupils appeared to have difficulty, the next stage of the study was to determine whether a period of teaching intervention would help to alleviate these problems. The teaching intervention took the form of a teaching experiment using teacher-directed activities in the form of worksheets. These worksheets were designed and selected based on the fact that the students appeared to be at the beginning of the procedural layer.

A second purpose of the study was to design a situation where the students would come face-to-face with their misconceptions and observe what they did under those circumstances. This situation took the form of a teaching experiment. The teaching experiment allowed the students' interaction, with each instructional step to be monitored closely. The important feature of the teaching experiment was that the researcher's task was to help the student build up a particular cognitive framework.

The teaching experiment was used by researchers in various studies. Post et al. (1985) investigated fourth-grade students' understanding of the order and equivalence of rational numbers in an eighteen-week teaching experiment. The instructional programme consisted of thirteen lessons, from three to eight days each. The children, working individually and as groups, were introduced to the part-whole interpretation of rational number by means of circular and rectangular pieces of laminated colored paper as well as other manipulative aids. Kantowski (1977) used a teaching experiment technique as the third phase of a study that was "to uncover information about, and gain a greater insight into, processes involved in solving complex, non-routine problems" (p. 164). This phase of her study lasted four months and included instruction in geometry using heuristic instructional techniques. A teaching experiment was also used by Kerslake (1986) to investigate whether a period of teaching intervention would help alleviate the difficulties that the pupils appeared to have with fractions. Bell, Swan & Taylor (1981) used a three phase study (exploratory interviews, a diagnostic test and the preparation and use of calculator enriched teaching materials) to examine the conceptual problems that children experienced when attempting to solve verbal problems which contained decimal numbers. The exploratory interviews were carried out:

> ... to discover in sufficient detail the conceptual systems with which pupils approached the set of tasks in question, and to indicate the suitability of the intended teaching procedures. (p. 403)

The diagnostic test was designed on the results of the interviews and administered:

> ... to determine how extensively the various misconceptions identified in the interviews exist among the group to be taught. (p. 403)

The preparation and use of the teaching material, with pre- and post-testing, was designed:

> ... to remedy the identified misconceptions, by activities-problems, exercises and games-in which the misconceptions are exposed and discussion of the resulting conflict leads to their correction. (p. 403)

The results showed that the teaching was effective as there was a marked improvement in understanding decimal place value. The results also indicated that direct work on choice of operation was needed as well as on the underlying sub-concepts.

The teaching experiment seemed to be a means by which students can be helped to construct a knowledge framework to deal with the handling of the problems in question. The teaching experiment allowed students' interaction, with each instructional step monitored as closely as possible as the treatment proceeded.

Because the methodology of a teaching experiment required detailed monitoring of the learning situation, the experiment was conducted with a small group. Eight students volunteered to be part of the teaching experiment. The remainder of the Math 15 class stayed with the regular teacher but worked through the same worksheets as the subjects. All worksheets that were prepared by the researcher for the experiment were passed on to the regular teacher to use with the remaining students as he saw fit. Both groups had dealt with the topic of decimals prior to Math 15.

The curriculum for the teaching experiment featured experiences which highlighted various decimal number subconstructs and the use of decimal language in settings where the students could validate such knowledge against some elements of reality (Hiebert & Wearne, 1987; Strefland, 1987; Kieren, 1987; Kerslake, 1986). The researcher wanted the students to feel at ease during the experiment. The classroom thus was equipped with tables and chairs rather than with individual desks, allowing the students to work together with ease. The students generally worked in groups, although one student preferred to work alone and shared his comments and reflections with the teacher/researcher. The teaching experiment did not involve any direct teaching by the researcher and used the conflict teaching approach (Swan, 1983). The students were encouraged to make, discuss and reflect on their own misconceptions and errors by means of worksheets. The worksheet approach was chosen on the basis of successful techniques used elsewhere (Swan, 1983; Kerslake, 1986). Worksheets were also used by Kerslake (1986) as part of her teaching experiment to investigate whether a period of teaching intervention would help alleviate the difficulties that the pupils appeared to have with fractions.

The worksheets contained potentially meaningful situations and gave the students the opportunity to work together and discuss each situation. Each worksheet involved experiences and situations that the students had encountered in previous years in the study of mathematics. The students worked together on the worksheets, discussed their responses with each other and commented on each other's work. The purpose of the discussion among the students was to involve the students in reflecting on their own misconceptions and errors, and to encourage them to reflect, debate and resolve any inconsistencies revealed by different answers. There were times when the teacher/researcher was able to be of some assistance in the discussion. The teaching experiment allowed for immediate feedback for the students which enabled the students to evaluate their own performance and correct mistakes as they arose. There was therefore no need for the teacher/researcher to mark and return students' work although the worksheets were kept on file by the teacher/researcher.

During the class time when the students were completing the worksheets, the teacher/researcher circulated from one table to another and made field notes and kept a record of any relevant comments and actions. The field notes contained information on: the mood of the students, their frustration level, how they related to the rest of their group, how the group related to each other as a group and as individuals, how one group related to another group, how they related to the worksheets, and any other factor that was at work that day.

The teaching experiment did not happen exactly as planned. The teacher/researcher had the following expectations of the teaching experiment. It was felt that the:

1) students would work in groups,
2) students would work as a group on the worksheets,
3) students would discuss each question before answering the question,
4) students would realize that they did not share the same meaning for each question,
5) students would discuss the various meanings held by different members of the group,
6) students would stay on task for the majority of the time,
7) discussions within the groups would provide insight about the thinking process of the students,
8) discussions would help explain the various misconceptions,
9) insights provided by the discussions would give a clear indication of what

could be done to help alleviate some of the misconceptions,

10) teacher/researcher would go from group to group making notes on the discussions,

11) teacher/researcher would take part in some of the discussions to probe for further information about the thinking process,

12) teacher/researcher would collect the worksheets and make note of any misconceptions that had changed or been alleviated,

13) teacher/researcher would accept all interpretations given by the students, and

14) teaching experiment would occur in a relaxed atmosphere.

There were slight differences between the planned teaching experiment and the actual teaching experiment. The students did work in groups but did not work as a group. Each member of the group worked individually on the worksheet and then those individuals compared answers. The discussions which took place were primarily to ask how to do a certain question or to compare answers and did not give in depth insights into the students' thinking processes; the discussions did provide explanations concerning the various misconceptions presented in the pretest. The insights provided by the discussions showed that reteaching correct procedures would help alleviate some of the procedural misconceptions. More research on this topic would present information on how to alleviate some of the conceptual misconceptions. However, the teaching experiment did provide insight about the characteristics of the students themselves and about how they coped with mathematics. Knowing how these students coped with mathematics helped explain why these students were experiencing difficulties with the decimal number system.

The teacher/researcher made field notes on a daily basis of the interactions that took place within the groups and between the groups, and any other factors at work that day. The students expected that the teacher/researcher would give them all the necessary correct procedures to complete the worksheets. In some instances, some students became frustrated when the teacher/researcher would join the group and ask probing questions.

Although there were some discrepancies between the teaching experiment as planned and what actually took place, the purpose of the teaching experiment was to observe what kind of reconstruction would occur under certain instructional conditions. It was felt by the researcher that the discrepancies did not invalidate the goal of the study and that what actually took place was valid as growth did occur.

The Post-test and Delayed Post-test

At the conclusion of the teaching experiment a post-test (Appendix C) was administered. The post-test results were to be compared with the pretest results to determine whether the teaching experiment alleviated some of the problems the subjects displayed on the pretest. The post-test would show whether there was an increase (or decrease) in students' knowledge. A delayed post-test (Appendix D) was administered after a further ten weeks, during which time the students did not receive any additional instruction on decimals. The delayed post-test consisted of the questions that were identical on the pretest and the post-test. The results were again compared to the results of the pretest and post-test. The delayed post-test would confirm the increase (or decrease) in students' procedural knowledge.

Delayed Student Interviews

A subsample of students was chosen on a volunteer basis to be interviewed after the delayed post-test. The interviews took place the first week of April, 1989, one month after the delayed post-test. The purpose of the delayed interviews was to determine whether the conceptual knowledge of the students had changed.

Data Analysis

Throughout the study a three-stage analysis scheme was used following that used by Pothier & Sawada (1983).

Immersion stage: The first stage involved a continuous process of analysis characterized by an intensive focus on the students' behavior. The flexible nature of the interviews and the teaching experiment provided the direction for immediate probing and variation in the problems. Field notes were recorded daily.

Reflection stage: During this period of data collecting, time was devoted to reflecting upon the students' responses and upon the interactions taking place between the students.

Documentation stage: The third stage of the analysis was conducted after the data had been collected. All sources were reviewed and tabulated.

Charts, and tables were constructed for the purpose of comparing and interpreting the results and for gaining further insights. These interpretations are presented in Chapter IV.

Summary

In order to meet the objectives of this study various procedures were used: a pretest, student interviews, a teaching experiment, a post-test, a delayed post-test and delayed student interviews. The pretest and pretest student interviews were used to ascertain the decimal number knowledge of the students. On the basis of this information, worksheets were prepared for use in the teaching experiment. The teaching experiment did not involve any direct teaching by the teacher/researcher. The worksheets were used as a means of getting the students to deal with their misconceptions about decimals and to discuss and reflect on these errors and misconceptions. During the teaching experiment the teacher/ researcher reflected on the students' responses and on the interactions taking place among the students. A post-test was administered after the teaching experiment to see if some of the misconceptions had changed or been alleviated. A delayed post-test was administered ten weeks after the post-test and the results were compared to the pretest and the post-test to determine whether there has been any increase in the students' procedural knowledge. The students were interviewed one month after the delayed post-test to determine whether their conceptual knowledge had changed.

The experience of data collection and simultaneous analysis was demanding; the results were the interpretations of the author. The experience resulted in insights into students' perceptions of decimal number knowledge and ways in which students function within the world of decimal numbers. The insights formed part of the final interpretations.

CHAPTER IV

Results of the Pre-Experimental Testing and Interviews

Introduction

One purpose of this study was to ascertain decimal knowledge of students enrolled in Math 15. This knowledge was sought through a pretest and interviews pertaining to questions on the pretest. The pretest (Appendix A) was administered to twenty-nine students and the pretest interviews were conducted with students selected on a volunteer basis. The pretest was used to expose and identify misconceptions that the students had already formed about the topic of decimal numbers.

Past research led to the assumption that these students would be using procedural techniques when they attempted to solve decimal computation problems. The nature of the Math 15 students would also lead to the assumption that they would manipulate symbols according to syntactic rules that they had previously been taught.

In this chapter, the pretest and the pretest interviews were analyzed to ascertain the decimal knowledge of the students. This information was also used to prepare worksheets for the teaching experiment.

Decimal Knowledge of the Students

To ascertain the decimal knowledge of the students the results of the pretest and the pretest interviews were analyzed under the following headings:

	Related test items
a) verbalising a decimal	1
b) place value	3,4,8,9
c) estimation	10
d) operations	7,11,12,17 18,22,23
e) scale reading	5,6
f) using zero as a place holder	5d,5e,6c,13e,14, 15b,22c,22d
g) comparing decimals	2,13,16
h) denseness of decimals	15
i) problem solving	19
j) fraction-to-decimal translation	21,20

Most of the items on the pretest appeared on Swan's (1983, 1988) Decimals Diagnostic Test. The researcher modified Swan's test to include place value, estimation, operations, problem solving and fraction-to-decimal translation. It was felt by the researcher that these topics and those presented on Swan's test would give a wider range of the decimal knowledge held by the students. Each heading was presented separately.

The pretest interviews were conducted after the pretest and before the teaching experiment (Appendix B). As this was a new experience for these students, the majority did not want to be interviewed alone but wanted friends with them. The interviewer agreed to this. Some interviews, therefore, involved individual students, and some involved two students. It was felt by the interviewer that the interviews which involved more than one student presented a different picture. Most often one student was reacting to the response given by the other student. This was especially true if one student felt that the other student was better at math and therefore the other student's responses were felt to be correct. These interviews showed what one student was thinking and how a second student responded to that. Parts of the pretest interviews are mentioned throughout this chapter where applicable.

Verbalising a Decimal

Item 1 (Figure 4:1) revealed that slightly over half (52%) of the students correctly chose nine and seven tenths as being the correct response to the meaning of 9.7 (Figure 4:1). Seven students (24%) did not interpret the meaning of 9.7 but chose a written verbalisation of 9.7 such as nine point seven or nine decimal seven.

Place Value

In general, the pretest revealed that the majority of students could name the place values of a decimal number. The pretest revealed that it was easier for the students when a decimal number was given and they were to indicate the place value of a specified digit within that number.

Item 4 (Figure 4:2) gave the student the decimal number 0.248 and asked what the two stands for. The majority of students (19 or 66%) gave the correct response.

Item 3 (Figure 4:2) asked the students to choose the response that represented thirty-seven thousandths. Approximately one third of the group (10 or 34%) gave the correct response. Nine students (31%) responded with 37 000 which contained no decimal point at all. It seemed that a number of students were interpreting decimals as whole numbers.

Item 8 (Figure 4:2) asked the students what four tenths was in hundredths. Very

Figure 4:1
Verbalising a Decimal
(N = 29)

---

1. What does 9.7 mean?

Check (✓) the best answer from the following:

a) Ninety seven (2)

b) Nine remainder seven (3)

c) Nine hundred and seven

d) Nine and a seventh (2)

[✓ e)] Nine and seven tenths (15)

f) None of these, I think it means (7) ____________

Nine point seven or Nine decimal seven

Figure 4:2
Place Value
(N = 29)

3. Which of the following is thirty-seven thousandths?

[✓ a)] 0.037 (10)

b) 0.37 (5)

c) 37

d) 37000 (9)

e) None of these. I think the answer is (5).

4. In the decimal number 0.248 the 2 stands for tenths (19).

8. Four tenths is the same as 40 hundredths. (6)

9. Six tenths as a decimal is 0.6. How would you write as a decimal three hundredths? .03 (21)

few students (6 or 21%) gave the correct response. They were unable to interpret one place value as another place value.

The majority of students (72%) gave the correct response to item 9 (Figure 4:2). Item 9 required that the students write three hundredths as a decimal.

The results of items, 3, 4, 8, and 9 were summarized in Figure 4:2.

Estimation

The following question (item 10) was asked on the pretest:

Estimate the answer then place the decimal point in the given answer.

a) 2.42 x 3.610 = 08736200
b) 3.20 ÷ .08 = 0040000
c) .42 x .23 = 00096600

A very small number of the students gave the correct answer. Table 4:1 showed the question, the number of students who responded with the correct answer and the number of students who gave the most common response. It can be seen in Table 4:1 that the majority of students did not give the correct answer. By examining the most common response it seems to suggest that a certain procedure was used. The students appear to have counted from the right, the number of places equal to the sum of the number of digits to the right of the decimal point in the problem and inserted the decimal point there. In Table 4:1 it can be seen that the majority of students used the procedure of counting places with both the multiplication and division problems.

In order to verify this hypothesis a sample of the students who wrote the pretest was interviewed. The interview questions dealt with problems that were on the pretest. The students were given a clean copy of the question and asked to place the decimal point in the answer. After the students placed the decimal point where they thought appropriate, the researcher asked each student, "How do you know that it goes there?" The students who gave the most-common response answered that they had counted places because that was what they were supposed to do. The answer for the division problem was the same; they counted places.

Hiebert & Wearne (1984) indicated that there were three sites in the process of computing with decimal numbers that demarcate the primary sources of students' difficulty. At site 1 many students did not know what the symbols meant; at site 2 many students did not know why the computational procedure worked, and at site 3, many students were not aware that answers should be reasonable. It appeared that the majority

Table 4:1
Estimation Responses and Most Common Responses
(N = 29)

Question	Correct Response	N	Most Common Response	N
a) 2.42 x 3.610 = 08736200	8.7362	4	87.362	22
b) 3.20 ÷ .08 = 0040000	40.0	3	4.0	18
c) .42 x .23 = 00096600	0.0966	3	9.66	21

of students who wrote the pretest failed to connect the decimal symbols with meaningful referents (site 1); they recalled and applied a memorized rule for which they seemed to have no rationale (site 2), and they were unaware that the answers given were unreasonable (site 3). The interviews revealed that the students' main concern was applying a procedure. After placing the decimal point, the student did not verify whether or not the answer was reasonable.

The following is a short excerpt from one of the interviews.

(I: Interviewer, S: Student)
I: Are you sure that the decimal point goes there?
S: Yes.
I: How do you know that it goes there?
S: All you have to do when you multiply decimals is count places.

The student was able to see that the answer was incorrect after the teacher/researcher worked with that individual. The student then realized that extra zeros had been added to the real answer. Detailed interview transcripts are presented in Appendix B.

Operations

The performance on the items that dealt with addition, subtraction, multiplication and division of decimal numbers was quite well done. Figure 4:3 summarized the results of questions 11, 12, 17, 18, 22, and 23. The directions for question 7, also dealing with operations, were misread by the majority of the students and therefore the results from it were not used for the pretest analysis.

i) Addition and subtraction. The majority of students gave the correct response for the items dealing with addition and subtraction of decimal numbers except for item 11c (Figure 4:3). The most common response (52%) for item 11c was .83. This seems to indicate that these students did not follow the procedure for the subtraction of decimal numbers. These students may have used decimal numbers as whole numbers.

Item 11c was one of the questions asked during the interview. Of the nine students who were interviewed after the pretest, only two said that the tenths were subtracted. The interviews provided interesting insights into how these students subtracted decimal numbers. There was no difficulty in subtracting two decimal numbers that had the same number of digits before and after the decimal point (example: 4.54 - 2.44). However, there was difficulty when a decimal number with one digit after the decimal point was subtracted from a decimal number having two digits after the decimal point (e.g., .86 - .3).

Figure 4:3
Operations
(N = 29)

11. Perform the following subtractions.

a) .78 - .35 = .43 (24)
b) .60 - .36 = .24 (23)
c) .86 - .3 = .56 (10)

---

12. Perform the following multiplications.

a) 6 x .4 = 2.4 (18)
b) 2 x 3.12 = 6.24 (22)
c) 8 x .06 = .48 (13)

---

17. Perform the following additions.

a) 4.6 + 2.3 = 6.9 (27)
b) 5.3 + 2.42 = 7.72 (25)
c) 5.1 + .46 = 5.56 (22)

---

18. Perform the following divisions.

a) 2.1 ÷ 3 = 0.7 (15)
b) .24 ÷ .03 = 8 (11)
c) .56 ÷ 7 = .08 (6)

---

22. Write down the next 2 numbers in each sequence as decimals.

a) 0.2, 0.4, 0.6, 0.8 , 1 (adding on 0.2 each time) (7)
b) 0.3, 0.6, 0.9, 1.2 , 1.5 (adding on 0.3 each time) (6)
c) 0.92, 0.94, 0.96, 0.98, 1 , 1.02 (adding on 0.02 each time) (13)
d) 1.13, 1.12, 1.11, 1.10 , 1.09 (taking away 0.01 each time) (15)

---

23. a) Add 0.1 to 4.256 4.356 (22) d) Take away 0.1 from 15.835 15.735 (19)

b) Add 0.1 to 3.9 4 (20) e) Take away 0.1 from 13 12.9 (11)

One student who was interviewed said that the answer to the question .86 - .3 was .53. However, he did not treat the two decimal numbers as whole numbers. He lined up .3 under .86 with the 3 underthe 6 and subtracted. He subtracted the 3 from the 6 and the 3 from the 8 to give him an answer of 53. To place the decimal, he counted places. He said he had to count three places because there was a total of three decimal places in the two decimal numbers. He placed a decimal point after the 3 and counted that as one decimal place so three decimal places brought him to .53 (Figure 4:4A).

Two students said that the answer was .83 but used different procedures. One student changed .3 to .03 to have the same number of digits after the decimal point in both numbers and then subtracted (Figure 4:4B). When asked if .03 was the same as .3 he said that it was because, "you can add zeros to decimals and it doesn't change anything."

The second student who responded with .83 placed .3 under .86 with the 3 under the 6 and subtracted to give her 83. She then said that she had to count two decimal places and placed the decimal point in front of the 8 to give her an answer of .83 (Figure 4:4C). When asked why she had to move the decimal point two places, she replied, "because .86 has two decimal places and .3 has only one decimal place so you move the decimal point the same number of places that is in the bigger number."

Lining up the 3 under the 6 was common among the 9 students interviewed. Another student responded with .083. She subtracted 3 from 86 and then counted three places (Figure 4:4D). She said she counted three decimal places because the two decimal numbers had a total of three decimal places.

An interesting case involved the student who said the answer was 5.3. She lined up the two decimal numbers with the 3 under the 6 (Figure 4:4E) and then subtracted the 3 from the 6 and the 3 from the 8 to give 53. She then placed the decimal point between the 5 and the 3. When asked why she had placed the decimal point between the 5 and the 3 she replied, "I just looked directly up and put the decimal." Her actions showed that she had looked up at the nearest decimal number (.3) and placed the decimal in her answer directly below the decimal point in .3.

ii) Multiplication and division. The results for the multiplication and division questions were summarized in Figure 4:3. The interviews revealed that the students knew that they could multiply and then count places to solve a multiplication problem; however, the students' process of counting decimal places was not always consistent. Figure 4:5 outlines two procedures that the students used when counting decimal places. One student did not count a space occupied with a zero as a countable space. When asked to multiply .05 and .4 she responded with .20. When asked how she had arrived at .20 she replied, "I multiplied the 5 and the 4 which makes 20 and then I counted two decimal places to make .20" (Figure 4:5A). When questioned further about the two

Figure 4:4
Subtraction Examples

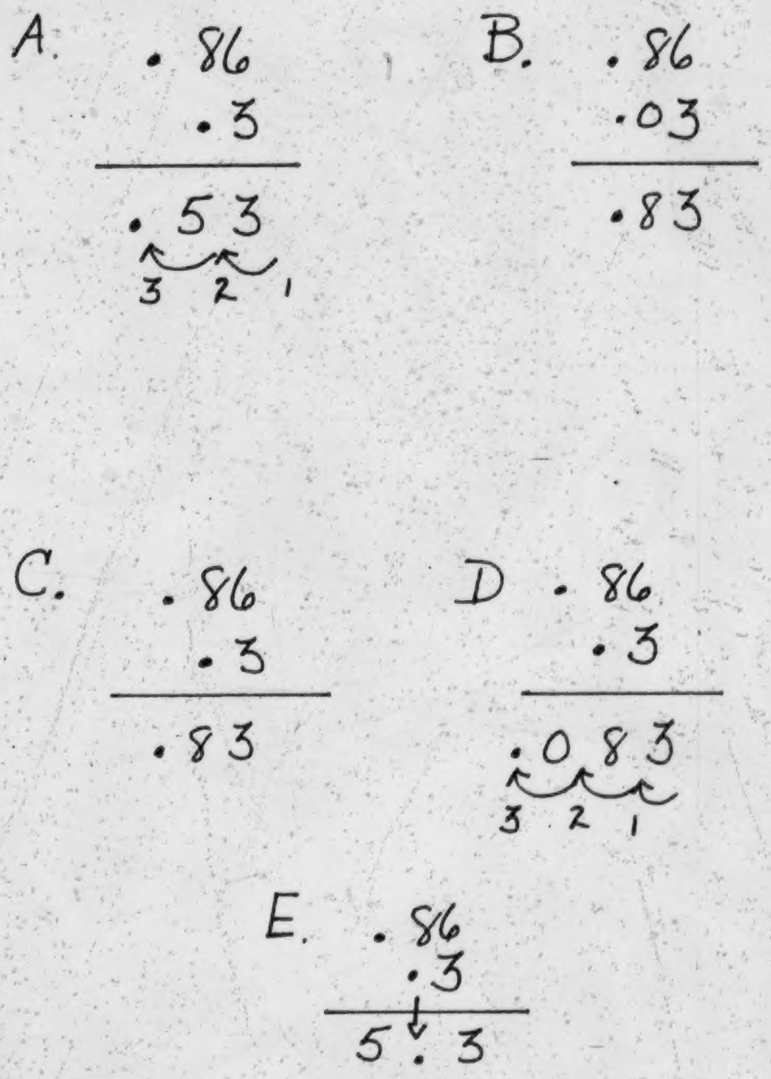
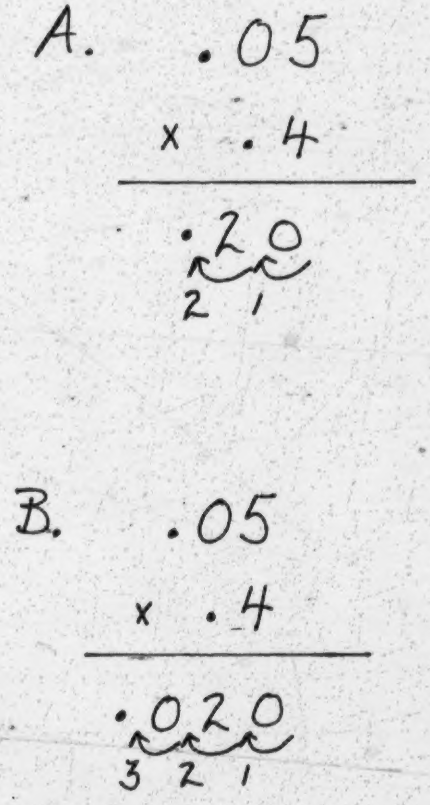

A. .86
.3
.53
3 2 1
B. .86
.03
.83
C. .86
.3
.83
D .86
.3
.083
3 2 1
E. .86
.3
5.3

Figure 4:5
Multiplication Procedures


A.
.05
x .4
.20
2 1
B.
.05
x .4
.020
3 2 1

decimal places she said, "Well, . 4 has one decimal place and although .05 has two places you only count one place because you don't count the zero place." Other students counted decimal places as in Figure 4:5B.

The performance on the division question (question 18) was poorer than on the multiplication question (Figure 4:3). Of the twenty-nine students who wrote the pretest, eight did not respond to the division question. It is not known why they did not respond. The responses to question 18 revealed that the students were concerned with the counting of decimal places. The majority of incorrect responses had the correct digit but the decimal point was misplaced.

The interview data revealed that the students found division of decimal numbers to be the most difficult of the four operations. They knew that there was a rule that used the counting of decimal places but only two of the nine students interviewed knew what the procedure was and how to use it.

Scale Reading

In general, the pretest revealed that the students could read scales subdivided into tenths, but they were less successful at reading scales marked in hundredths and fifths. They had trouble estimating a reading which lay between marked calibrations. Items 5 and 6 dealt with scale reading. Items 5g and 5h (Figure 4:6) were divided into fifths and students in general did poorly on these items. The numbers inside the parenthesis in Figure 4:6 revealed the number of students who gave the correct response to each item on question 5. Eleven students responded with 2.9 for item 5g and eighteen students responded with 5.2 for item 5h. This indicated that there was no notion of the idea of ten-ness of a decimal for those pupils. The majority of students did well on items 5a, 5c where the scales were marked in tenths, and 5i, estimation. In items 5d and 5e students had to supply zeros where the zero was used as a place holder. The most common response to item 5d was 8.1 and the most common response for item 5e was 2.3. Item 5b required that the students estimate a reading which was between marked calibrations. Very few students gave the correct response. The most common response was 2.8. Three students responded with 2.8 1/2. These students realized that the reading was halfway between 2.8 and 2.9, but were unable to express it as a decimal number. Item 5f posed a problem because it was divided into hundredths rather than tenths.

Question 6 (Figure 4:7) also revealed that the students could read scales subdivided into tenths more easily than scales marked in hundredths or fifths. Items 6a and 6b were answered correctly by the majority of the students although item 6b, which was subdivided into hundredths, had fewer correct responses than item 6a which was divided into tenths.

Figure 4:6
Scale Reading
(N = 29)

5. Read the following scales and write your answers in the boxes provided. Give all your answers as DECIMALS.

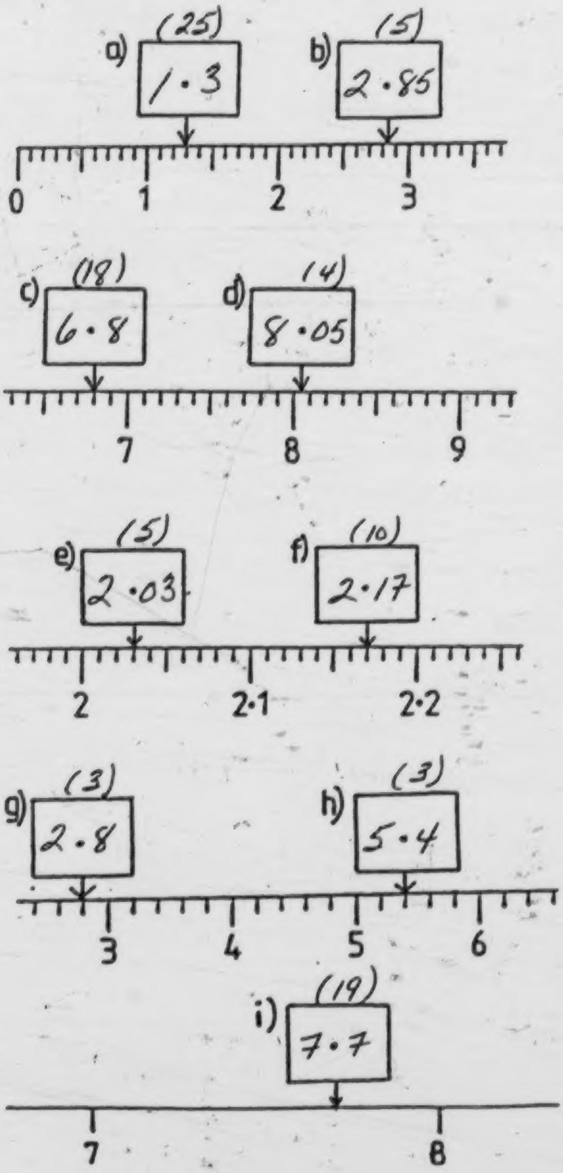
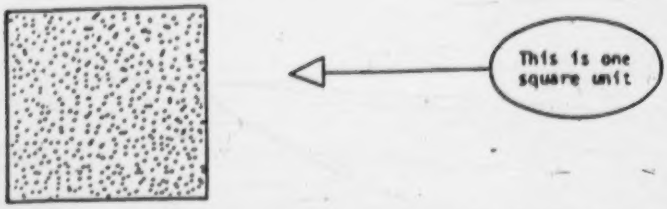
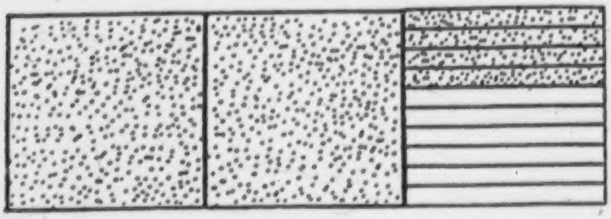

(25)
a)
1·3
(5)
b)
2·85
0
1
2
3
(18)
c)
6·8
(4)
d)
8·05
7
8
9
(5)
e)
2·03
(10)
f)
2·17
2
2·1
2·2
(3)
g)
2·8
(3)
h)
5·4
3
4
5
6
(19)
i)
7·7
7
8

Figure 4:7
Scale Reading
(N = 29)



6. Give your answers as DECIMALS

a)

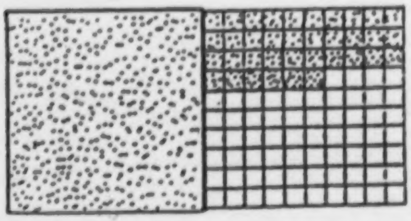


The area shaded

is 2 • 4 (23)

square units

b)

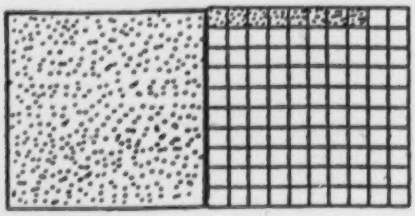


The area shaded

(18)

is 1 • 36 square units

c)

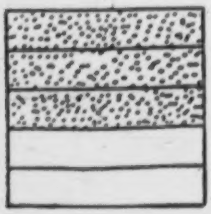


The area shaded

(6)

is 1 • 08 square units

d)



The area shaded

(3)

is 0 • 6 square units

The majority of students responded with 1.8 for item 6c where zero was used as a place holder. Item 6c was subdivided into fifths and the majority of students responded with the answer of 0.3.

Using Zero as a Place Holder

These questions were classified into two types, those for which the students had to supply the zero themselves, and those where the zeros were already provided and the students were required to interpret their meaning.

i) Questions for which students have to supply zeros 5d,5e (Figure 4:6); 6c (Figure 4:7); 15b,22c,22d (Figure 4:8).

Table 4:2 summarized the number of correct responses to the related test items. The small number of correct responses on items 5d,5e, and 6c coincided with the students' inability to read scales which were subdivided into units other than tenths. Item 15b revealed the inability of the students to generate decimals which lay within given intervals.

Items 22c and 22d, presented a different picture. With these items, the majority of students gave the correct response (Table 4:2). The students seemed to have more confidence when dealing with straightforward operational procedures and succeeded quite well on the straightforward operation questions.

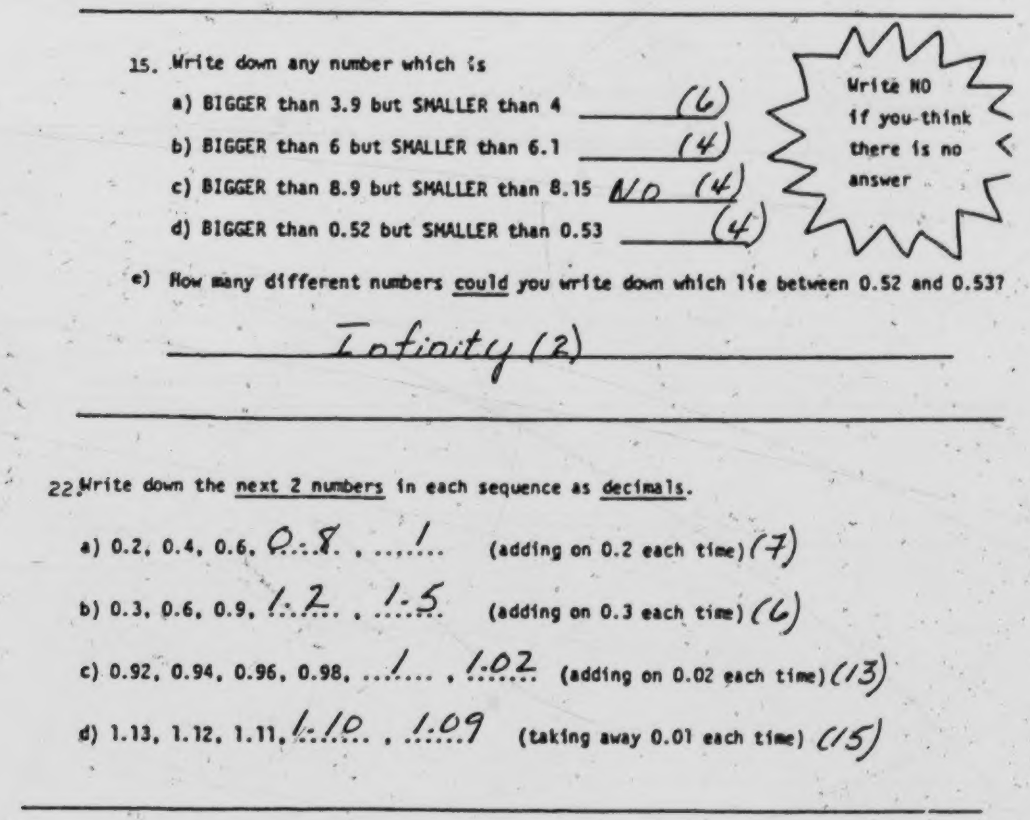
ii) Questions for which pupils have to interpret zeros (13e,14)

The results for questions 13e and 14 were summarized in Figure 4:9. The students performed quite well on these two questions.

Comparing Decimals

Questions 2, 13 and 16 are similar and required the largest number to be chosen from a group of decimals. Figure 4:10 summarized the results of these three questions. Question 2 required the students to choose the largest number from a group of three decimals with different lengths (number of digits after the decimal point). Only eight students gave the correct response. Figure 4:10 showed the number and the percentage of students who gave each response. The most common response was 0.236. Most of the students who chose 0.236 as being the largest of the three numbers said that it was the largest because it had the most number of digits after the decimal. Some of the students were treating the decimal as a whole number. They reasoned that 236 was bigger than four and bigger than sixty-two. Most of the students who chose 0.4 as being the largest of the three numbers said that it was the largest because it only had tenths. A few students responded by saying, "Tenths are bigger than hundredths and bigger than

Figure 4:8
Using Zero as a Place Holder
(N = 29)



15. Write down any number which is

a) BIGGER than 3.9 but SMALLER than 4 ______ (6)
b) BIGGER than 6 but SMALLER than 6.1 ______ (4)
c) BIGGER than 8.9 but SMALLER than 8.15 NO (4)
d) BIGGER than 0.52 but SMALLER than 0.53 ______ (4)

Write NO if you think there is no answer

e) How many different numbers could you write down which lie between 0.52 and 0.53?

Infinity (2)

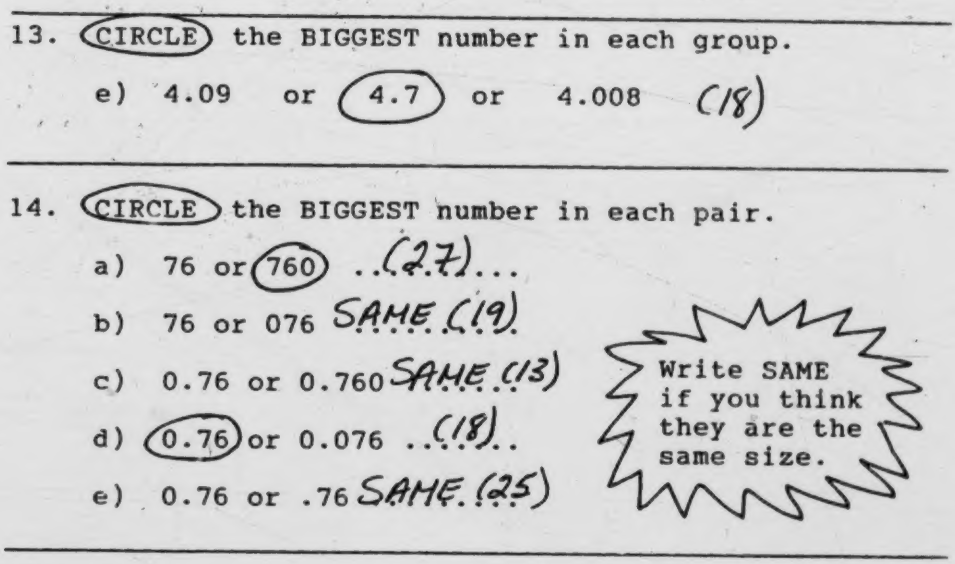
22. Write down the next 2 numbers in each sequence as decimals.

a) 0.2, 0.4, 0.6, 0.8, 1 (adding on 0.2 each time) (7)
b) 0.3, 0.6, 0.9, 1.2, 1.5 (adding on 0.3 each time) (6)
c) 0.92, 0.94, 0.96, 0.98, 1, 1.02 (adding on 0.02 each time) (13)
d) 1.13, 1.12, 1.11, 1.10, 1.09 (taking away 0.01 each time) (15)

Table 4:2
Using Zero as a Place Holder
(N = 29)

Question	**Correct Response**	**N**
5d	8.05	4
5e	2.03	5
6c	1.08	6
15b	$\geq$ 6.01 and $\leq$ 6.09	4
22c	1, 1.02	13
22d	1.10, 1.09	15

Figure 4:9
Using Zero as a Place Holder
(N = 29)



13. CIRCLE the BIGGEST number in each group.

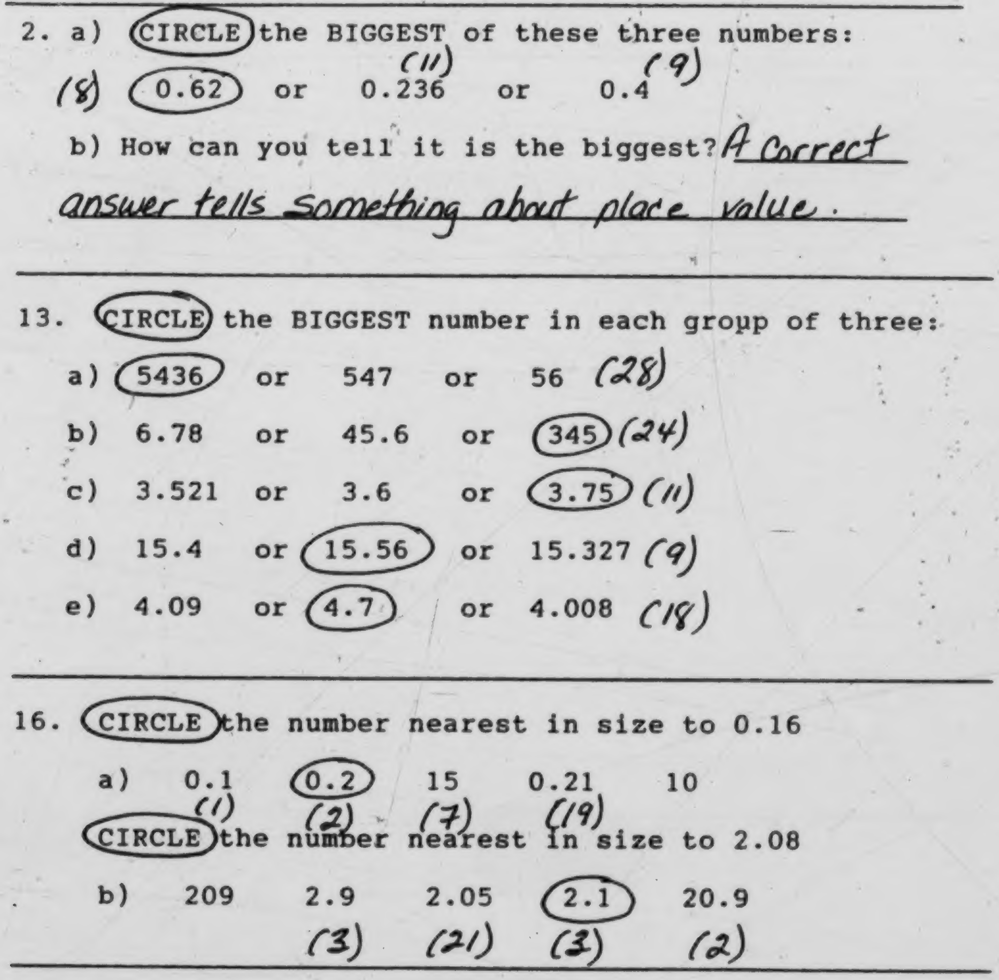
e) 4.09 or [✓ 4.7] or 4.008 (18)

14. CIRCLE the BIGGEST number in each pair.

a) 76 or [✓ 760] ...(27)...
b) 76 or 076 SAME (19).
c) 0.76 or 0.760 SAME (13)
d) [✓ 0.76] or 0.076 ...(18)..
e) 0.76 or .76 SAME (25)

Write SAME if you think they are the same size.

Figure 4:10
Comparing Decimals
(N = 29)



2. a) CIRCLE the BIGGEST of these three numbers:
(8) [✓ 0.62] or (11) 0.236 or 0.4 (9)

b) How can you tell it is the biggest? A correct answer tells something about place value.

13. CIRCLE the BIGGEST number in each group of three:
a) [✓ 5436] or 547 or 56 (28)
b) 6.78 or 45.6 or [✓ 345] (24)
c) 3.521 or 3.6 or [✓ 3.75] (11)
d) 15.4 or [✓ 15.56] or 15.327 (9)
e) 4.09 or [✓ 4.7] or 4.008 (18)

16. CIRCLE the number nearest in size to 0.16
a) 0.1 (1) [✓ 0.2] (2) 15 (7) 0.21 (19) 10

CIRCLE the number nearest in size to 2.08
b) 209 2.9 (3) 2.05 (21) [✓ 2.1] (3) 20.9 (2)

thousandths, so 0.4 is the bigger number."

Within item 13, 13d posed the greatest difficulty with only nine students giving the correct response. The most common response for item 13d was 15.327. It appeared that the decimal numbers have been treated as whole numbers where thousands are bigger than tens.

Question 16 did not give good results. A very small number gave the correct response to items 16a and 16b. The most common distracter for item 16a was 0.21 whereas in item 16b it was 2.05.

Denseness of Decimals

Question 15 (Figure 4:11) tested the ability of students to generate decimals which lay within given intervals, and the fact that within any interval there existed an infinite number of decimals. The results reveal that very few students were capable of generating decimals which lay within given intervals.

The majority of students (more than 50%) responded with an answer of NO to items 15a, 15b and 15d (Table 4:3).

The results for item 15c showed that the majority of students were unable to resist the temptation to regard the decimal numbers as two independent whole numbers.

Only two students responded correctly to item 15e. One student gave the response of infinity and the second student gave the response of lots which was accepted as correct. The most common response was 'NONE' by 38% of the students. Three students said that there was only one number between 0.52 and 0.53. A number of students did not attempt an answer.

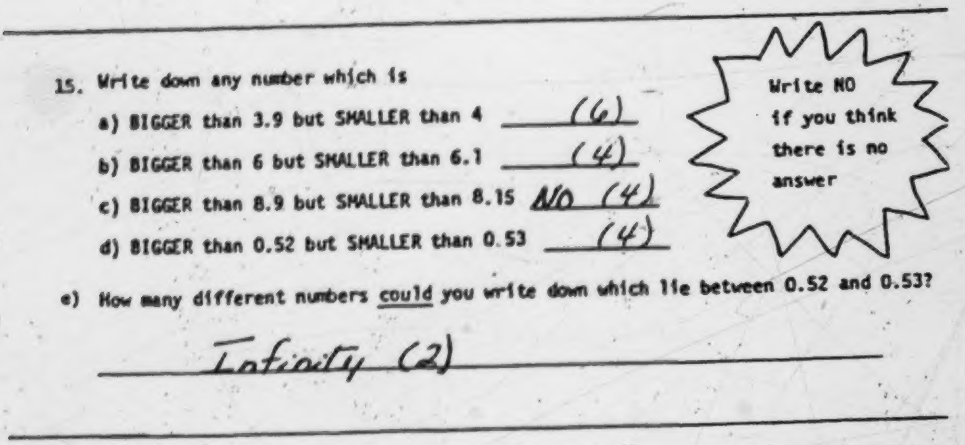
Problem Solving

Item 19 required that the students would choose the calculation that gave the correct answer to the question. Less than half the students listed the correct response. Ten students chose division operations rather than multiplication (Figure 4:12).

Fraction-to-Decimal Translation

Except for the fraction 1/2, students did poorly on the fraction-to-decimal translations (items 20 and 21, Figure 4:13). The most common responses were syntactic translations. Students performing syntactic translations kept "both the

Figure 4:11
Denseness of Decimals
(N = 29)



15. Write down any number which is

a) BIGGER than 3.9 but SMALLER than 4 (6)

b) BIGGER than 6 but SMALLER than 6.1 (4)

c) BIGGER than 8.9 but SMALLER than 8.15 NO (4)

d) BIGGER than 0.52 but SMALLER than 0.53 (4)

e) How many different numbers could you write down which lie between 0.52 and 0.53?

Infinity (2)

Write NO if you think there is no answer

Table 4:3
Denseness of Decimals Most Common Responses
(N = 29)

Question	Most Common Response	N
15. a)	NO	18
b)	NO	15
c)	≥ 8.10 and ≤ 8.14	19
d)	NO	18
e)	NONE	11

Figure 4:12
Problem Solving
(N = 29)

---

19. Circle the CALCULATION you would need to find the answer to the following problem.

My car can go 42 km on each litre of gas. How many kilometers can I expect to travel on 11.37 litres of gas ?

a) 11.37 ÷ 42 [✓ d) 11.37 x 42] (11)
b) 42 - 11.37 e) 42 + 11.37
c) 11.37 - 42 f) 42 ÷ 11.37

Figure 4:13
Fraction-to-Decimal Translation
(N = 29)

---

20. Which decimal is equal to the fraction 1/5 ?

a) .15 (12)

[✓ b)] .2 (3)

c) $.5\dot{1}$ (8)

d) .51 (0)

e) I don't know. (6)

---

21. What are the decimal numbers for the following fractions ?

a) 1/2 0.5 (13)

b) 2/3 $0.\bar{6}$ (5)

c) 3/4 0.75 (9)

numerator and the denominator of the fraction and put a decimal in someplace" (Resnick et al., 1989, p. 23). For example, the fraction 1/5 became 1.5 or .15. Syntactic translations produced a number that had the appearance of a decimal but had no relation to the quantity expressed in the fraction. Table 4:4 showed the percentage of students with the correct response and the percentage of students responding with a syntactic translation.

The majority of students knew that the fraction 1/2 was 0.5 and 3/4 was 0.75. During the interview sessions the students were asked to translate 1/2 into a decimal. The majority of the students answered 0.5 and a very small number answered 1.2. The interviewer then asked the student, "How do you know that 1/2 is equal to 0.5?" The student answered, "It just is. Everyone knows that." One student said that 1/2 was equal to 1.5. He justified his answer this way: "One half is half of something so as a decimal 1.5 would be something and a half." The interviews showed that very few students could change a fraction to a decimal. A few students knew the procedure of dividing the "bottom into the top". The difficulty arose when the students were asked to translate into decimals the fractions that they did not know from memory; this led to syntactic translations.

Summary

The students taking the pretest displayed no conceptual knowledge of the decimal number system. Their knowledge about the decimal number system was restricted to the manipulation of symbols according to syntactic rules.

The analysis of the pretest and the pretest interviews indicated that the students who took part in this study possessed a hodgepodge of decimal knowledge. They used a variety of procedural techniques when solving decimal computational problems and tried to apply syntactic rules that they had previously been taught. Some rules were applied correctly, but the majority were applied incorrectly. When a rule was applied correctly, the students did not know why the computational procedure worked and they were unable to state whether the answer was reasonable or not. They did not seem to be aware that it was possible to apply a rule incorrectly or that the answer could be incorrect. When asked if the answer was correct, the reply was, "It has to be. I used the rule."

In some cases, there was confusion about when certain rules could be applied. For example, the multiplication of two decimal numbers became a matter of counting decimal places and moving the decimal point. This same rule was applied to the division of decimal numbers because students could not remember the rule for dividing decimal numbers. Addition and subtraction of decimal numbers posed no problem as the same rule of lining up the decimal points could be applied to both operations. Some students used the monetary system as a referent when adding/subtracting decimal

Table 4:4
Fraction-to-Decimal Translation
(N = 29)

	FRACTIONS			
RESPONSE	**1/5**	**1/2**	**2/3**	**3/4**
Correct	N=3 10%	N=13 45%	N=5 17%	N=9 31%
Syntactic Translation	N=12 41%	N=11 38%	N=14 48%	N=13 45%

numbers. The monetary system was the only referent to which the students could relate decimals.

For the students in this study, conceptual knowledge of the decimal number system did not exist. They performed decimal computational problems using previously learned procedures. Their procedural knowledge was not extensive either. They had previously memorized a variety of rules and procedures but when their memory failed, the procedures became distorted. Because the symbols had no meaning, they were unable to realize that some of the procedures that were being applied were incorrect and therefore they had no way of realizing whether a solution was reasonable or not.

It appeared that these students were at the procedural layer (the third layer) of the Onion Model. These students seemed to have arrived at this layer without peeling the concrete-objective layer and the operative-interpretive layer (the first two layers of the model). It was as if a probe had been inserted at the third layer. For the students in this study, their knowledge of the decimal number system existed solely within the procedural layer. They did not seem able to relate the procedures or the symbols to another layer. Therefore, for these students, the procedural layer was the only available one.

After the pretest and the pretest interviews, eight students volunteered to take part in the teaching experiment which is described in Chapter V.

CHAPTER V

The Teaching Experiment

Introduction

Having identified certain aspects of decimal number knowledge with which students appeared to have difficulty, the researcher's next stage was to investigate whether a period of teaching intervention would help to alleviate those problems, or, at least some of those problems. The difficulties seemed to stem from a use of recall instead of reconstruction; this resulted in the students relying on the strategy of searching their memories for a previously taught algorithm. Sometimes the search was successful, but often it was not. It was felt that the style of the teaching intervention should make it possible for pupils to discuss their incorrect, and correct, strategies and misconceptions. Therefore, the intervention took the form of a set of worksheets at which the students worked in small groups, with the researcher as observer and helper.

The teaching experiment was previously used by researchers (Post et al., 1985; Bell, Swan & Taylor, 1981; Kantowski, 1977; Kerlsake, 1986) to investigate students' understanding of rational numbers, to uncover information into processes involved when solving problems and to examine the conceptual problems that children experience when solving verbal problems.

The teaching experiment seems to be a means by which we can help students construct a knowledge framework that may help them overcome some of the difficulties they experience. The teaching experiment allowed the researcher to monitor as closely as possible the students' interaction with each instructional step.

The aim of the teaching experiment was twofold: 1) to gain further evidence concerning the procedures and processes used when working with decimal numbers, and 2) to observe any changes in their decimal knowledge. This was done by monitoring the students' behavior during the teaching experiment and by reflecting upon the responses from the worksheets. An important feature of the teaching experiment was that the researcher's task was to help the student build up a particular cognitive framework. Thus the researcher took on the role of teacher and, as such, helped the students with mathematics. The students helped the researcher by providing reflections about the actions involved in learning mathematics and, in particular, about responses to specifically designed teaching activities.

This study attempted to understand the rationale behind the procedures and processes in terms of students' conceptual framework, and to find the relative place of these procedures and processes on the Onion Model of Decimal Number Learning.

Outline of the Teaching Experiment

The teaching experiment was carried out with eight students enrolled in Math 15 at one of the Edmonton Public High Schools during the four weeks, from November 14 to December 8, 1988. The students and the teacher/researcher met every day from 08:15 to 09:15.

The teaching experiment was elaborated upon in Chapter III. An elaboration on the four areas of difficulty which emerged during the pretest and the pretest interviews follows:

1. Place value

The majority of students could name the place values of a given decimal number but had difficulty choosing the correct response to a written decimal number. The students displayed difficulty in ordering decimals because they could not correctly identify one decimal number as being larger or smaller than a second or third decimal number. The teaching experiment focused on the meaning of a decimal number in order to try to overcome this problem.

2. Scale reading

The teaching experiment dealt with the difficulties displayed by the students in the pretest. The difficulties were reading scales marked in units other than tenths (hundredths, fifths) and estimating a reading between marked calibrations.

3. Fraction-to-decimal translation

The students performed poorly on this section of the pretest. Except for the fractions 1/2 and 3/4, the most common responses were syntactic translations that produced a number that had the appearance of a decimal number but had no relation to the quantity expressed in the fraction. Even though the pupils knew that there was a relationship between fractions and decimals, they were unable to use knowledge of this relationship to translate fractions into decimals. The teaching experiment allowed the students to work with fractions and decimals together.

4. Operations

The students had no concept of what it meant to multiply or divide decimal numbers. They had no conceptual understanding of these two operations and their procedural knowledge had been memorized. However, their memory failed them when it came to the division of two decimal numbers so they applied the multiplication rule and counted decimal places. Addition and subtraction posed less of a problem since the same rule of lining up the decimal point could be applied. The teaching experiment looked at all four operations.

The teaching experiment did not involve any direct teaching by the researcher and took the form of worksheets. This was done to enable the teacher/researcher to monitor closely the activities of the students. It also meant that student responses to worksheets could provide a detailed written record of student activities when they were faced with specific learning tasks. The worksheets were comprised of questions, problems and activities dealing with the four areas of difficulty and were developed by the researcher on the basis of the pretest and the pretest interviews.

Worksheets were designed so that students could work together, discuss their responses and comment on each other's work. The purpose of the discussion among the students was to involve the students in discussing and reflecting on their own misconceptions and errors, and to encourage them to debate and resolve any inconsistencies revealed by different answers. There were times when the teacher/researcher was able to be of some assistance in the discussion. The teaching experiment allowed for immediate feedback which enabled the students to evaluate their own performance and correct mistakes as they arose.

The classroom consisted of tables and chairs rather than individual desks; this allowed the students to work together more easily. The students worked in groups although one student preferred to work alone and share his comments and reflections with the teacher/researcher.

During the class time when the students were completing the worksheets, the teacher/researcher circulated from one table to another, made field notes and kept a record of any relevant comments and actions. The field notes contained information on the mood of the students, their frustration level, how they related to the rest of their group, how the groups related to each other, how they related to the worksheets. Other factors that were at work that day were also noted. All interactions which took place during the teaching experiment between students and between students and the teacher/researcher were described in the field notes. Where reports of such interactions appear, these came from the field notes.

The Design of the Teaching Experiment

Having decided on the teaching style, the teacher/researcher created worksheets and activities for each area of difficulty.

The worksheets that dealt with place value (Appendix E) included the following activities:

(i) comparing decimals;

(ii) using the number line to order decimals;
(iii) using index cards to order decimals;
(iv) labelling the place value of specified digits in a decimal number;
(v) working with specified place values;
(vi) carrying out exercises on the abacus.

The objectives of the place value worksheets were to:
- enable students to compare correctly numbers with different numbers of digits,
- enable students to arrange several decimal numbers in order of size and to be able to do this using a number line,
- encourage students to develop and verbalise their own rules for comparing decimals without having to use a number line, and
- understand the place value of decimal numbers by the representation of decimal numbers on the abacus.

The worksheets concerned with scale reading (Appendix F) included:

(i) reading the amount of medication in a syringe;
(ii) marking as correct or incorrect the readings on various number lines subdivided into tenths, fifths, or twentieths;
(iii) reading scales subdivided into tenths, fifths, or twentieths.

The objective of the scale reading activities were to enable students to read and interpret scales which have been subdivided into tenths, fifths, or twentieths, and to interpolate successfully between marked calibrations.

The fraction-to-decimal translation worksheets (Appendix G) included:

(i) comparing fractions, decimals and percents in terms of "equal", "less than" or "greater than";
(ii) taking various fractions, decimals and percentages and placing each term in the proper column. The three columns were labelled "less than 1/4", "1/4", and "greater than 1/4";
(iii) completing a chart with a fraction and its equivalent decimal number and percent;
(iv) expressing percents as decimals, decimals as percents, and fractions as percents;
(v) solving problems using percents;
(vi) finding equivalencies using blank playing cards marked with fractions, decimals and percents.

The objectives of the fraction-to-decimal translation worksheets were to:
- help students learn equivalencies between, fractions, decimals and percents,
- enable students to change from one form to another,

- provide students with exercises that would allow them to work with all three forms,
- be able to use decimals when dealing with money,
- correct the misconception of syntactic translations.

The operation worksheets (Appendix H) included the following activities:

(i) finding a pattern that existed within a pyramid that would enable the student to complete the pyramid;
(ii) completing sequences;
(iii) solving problems using the operations of addition, subtraction, multiplication and division;
(iv) playing the game Barricade;
(v) playing the game Treasure Chest.

The objectives of the worksheets dealing with operations were to:

- provide the students with activities that allowed them to decide which operations should be performed,
- help students with the reading and understanding of written problems,
- correct the misconception that a decimal number was just two integers separated by a dot.

Procedure

The eight students who took part in the teaching experiment were separated from the remainder of the Math 15 class for the length of the teaching experiment.

The students were free to form their own groups and sat at whichever tables they pleased. The groups did not change during the teaching experiment. Students were allowed to complete the worksheets at their own paces and could talk freely and discuss openly among themselves or with other groups. During this time the teacher/researcher circulated from table to table paying attention to what was happening within each group, taking part in a discussion, and beginning discussions to try to correct misconceptions.

Results of the Teaching Experiment: General Observations

The students taking part in the teaching experiment appeared to be quite at ease within this new milieu. The fact that there were tables and chairs in the classroom made it easier for group work. The students formed themselves into three groups: one group of four, one group of three with one student working alone.

The students appeared to enjoy working from the worksheets and the style of the presentation caused no problems in that the students could socialize and do math at the same time.

Absenteeism and lateness were common traits of these students. The mathematics class periods in which the teaching experiment was conducted began at 08:15 every day. Perhaps owing to this early hour, a few of the students were always 10 to 20 minutes late. Upon speaking to the regular teacher I was informed that this was common practice for these students. The students who were in class at 08:15 would begin the worksheets and they would be joined by the others when they arrived.

On few occasions the group was not very interested and it was difficult to motivate them. However, after letting them vent their frustrations, they managed to complete part of the worksheets. Most of their frustrations dealt with the fact that they were not very interested in school.

The students liked group work and discussion but the discussions were not always related to the worksheets. Some of their discussions centered around social activities or events. It was noticed that the students completed their worksheets individually and then compared answers. If the answers among the group were not the same, the question of, "How did you get that?", was asked. The students were concerned about getting the right answer and the correct answer was what was important. If one student in the group was seen as being the most competent in mathematics, the other members of that group would copy that answer. They did not want an explanation of how that answer was arrived at.

These students were mostly interested in procedures. They would ask for assistance when they did not know how to find the correct answer and often asked, "Could you help me with this one? I don't know how to do it." Helping them meant showing them the procedure that would lead to a correct solution. When posing probing questions to help them generalize a method, they would get frustrated and say, "Just tell me how to do it. That's all I want to know."

The discussions among the group members consisted of comparing answers. Very little of the thinking process was discussed with other members of the group. Procedures were shared with the other group members. One girl in this study asked for assistance on many occasions; she often said, "Show me how to do this. I want to understand." Her idea of understanding seemed to be knowing when to apply a procedure so that she would get a correct answer.

Calculators were not used. It was felt that the use of calculators would not allow the teacher/researcher to observe the procedures and how they were performed.

Two games, Barricade and Treasure Chest (Appendix I), were used during the study. To make a move on the gameboard the student had to answer correctly a decimal number question. To make the game more interesting, trivia questions were included as well. The students enjoyed using the gameboard and answering the trivia questions but were less enthusiastic if they had to answer a decimal number question.

The use of concrete materials was not planned by the researcher. Owing to the fact that these were high school students with low self-esteem, the researcher did not want to bring into the teaching experiment models such as Dienes' blocks because the researcher did not want to run the risk of the students feeling like elementary children having to play with blocks. Although there was a strong attempt to test and teach with meaning, no concrete materials were used in the study. Some of the items had references to physical situations (the medical syringe, the abacus). Other physical situations came about as the result of a student's suggestion. For example, on the third day of the teaching experiment, two students mentioned that they would like to play cards. That evening, the researcher marked blank playing cards with fractions, decimals, and percentages (Figure 5:1). The objective was to recognize equivalent triplets. The students showed enthusiasm when using these cards and did well on the activity.

In terms of general interest and ease of use, the teaching experiment appeared to have been successful. However, the overall success of the teaching experiment depended on the analysis of the pretest, posttest and the delayed post-test.

Results of the Testing

Following the teaching experiment, a post-test was administered. The post-test scores were compared to the pretest scores to determine whether there was an increase in scores. Ten weeks after the teaching experiment, a delayed post-test was administered. Again, the scores on the delayed post-test were compared to the pretest and the post-test scores. The pretest contained twenty-one questions as did the post-test. Fifteen of the pretest questions were parallel to fifteen questions on the post-test. The pretest contained more questions on place value than did the post-test and the post-test contained more questions on problem solving than did the pretest. The delayed post-test was composed of fifteen questions selected from the post-test. Therefore there were fifteen questions that were parallel across the three tests. The fifteen parallel questions were subdivided into six categories: scale reading (2), operations (4), place value (5), problem solving (1), estimation (1), and fraction-to-decimal translation (2).

The pretest, the post-test and the delayed post-test were administered to the entire Math 15 class; the means of the three tests appear in Table 5:1. There was a gain from the pretest to the post-test and again to the delayed post-test. The overall gain, from 37.45% to 54.36%, was 16.91%. A different number of students wrote each test owing to absenteeism. The students were not told ahead of time when they would write the post-test or the delayed post-test.

Figure 5:1
Finding Equivalent Triplets

0.90	0.8	$\frac{1}{3}$	75%	10%
25%	$\frac{3}{5}$	$\frac{90}{100}$	100%	$\frac{4}{5}$
$\frac{1}{4}$	0.20	1.0	$\frac{5}{5}$	20%
0.1	90%	0.5	0.25	33.3%
$0.3\overline{3}$	0.6	0.75	80%	$\frac{1}{2}$
$\frac{3}{4}$	$\frac{1}{5}$	$\frac{1}{10}$	50%	60%

Table 5:1
General Overview of Results for all the Students in Math 15

	Pretest N = 29	Gain	Post-test N = 24	Gain	Delayed Post-test N = 22
Mean	37.45%	+9.8	47.25%	+7.11	54.36%

The means on the post-test and delayed post-test of the eight students taking part in the study appear in Table 5:2. The pretest mean for these students was not available because the students were granted anonymity and did not put their names on the pretest. After the teaching experiment, the students in the study felt comfortable enough with the researcher to put their names on the post-test and the delayed post-test. During the ten-week interval between the post-test and the delayed post-test, two of the students who had taken part in the study quit school. Therefore, only six students wrote the delayed post-test.

The following section contains a detailed item analysis of the items that were identical on the post-test and the delayed post-test.

Detailed Item Results

The results of individual items that focused on the four areas of difficulty are outlined below. The items are as they appeared on the delayed post-test. These results pertained to the eight students in the study. However, as the remaining Math 15 students also completed the worksheets in a manner in which the regular teacher saw fit, it seemed appropriate that these results be examined as well. The percentage of all Math 15 students with the correct response are shown in Appendix J. The performance of the eight students in the study is parallel to the performance of the entire Math 15 class, therefore suggesting that the eight students who volunteered were a good representation of Math 15 students.

Place Value

The place value items were those that required an interpretation and understanding of the place value of decimal numbers. Two of the place value items dealt with using place value in order to compare the size of decimal numbers.

The items were:

1. a) CIRCLE the SMALLEST of these three numbers:

0.6   0.06   0.006

b) How can you tell it is the smallest?

10. a) CIRCLE the number nearest in size to 0.14

0.1   0.2   15   0.21   10

Table 5:2

Results of Students in Study

	Pretest **N = 29**	**Post-test** **N = 8**	**Gain**	**Delayed Post-test** **N = 6**
Mean	37.45%	46.26%	+9.05	55.30%

b) CIRCLE the number nearest in size to 2.08

209 2.9 2.05 2.1 20.9

The percentage of students with the correct response was shown in Table 5:3.

The scores for item 1a increased markedly after the teaching experiment. However, the majority of students could not verbalize why 0.006 was the smallest of the three numbers. The most common response for item 1b, even after the teaching, was because it had more zeros after the decimal point. Very few students were able to verbalize the answer in terms relating to place value.

The students had less success with question 10. The most common response for item 10a was 0.21 and the most common response for item 10b was 2.05. One student, in the delayed interviews, put into words the rule that was being applied. She said, "You need the same number of places after the decimal point." This implied that 0.21 was the closest number to 0.14 because it was the only decimal number that had two digits after the decimal point. She also stated that 2.05 was closer to 2.08 than any of the other numbers because "you have to have a zero after the decimal point." Very few students used place value interpretations to answer this question. They used visual clues rather than knowledge about decimal numbers. One student managed to cope quite well with the ordering of decimal numbers by referring to money when that individual saw a decimal number.

Questions 2 and 3 required the knowledge of specific place values within a decimal number.

The items were:

2. Which of the following is twenty-eight thousandths?

   a) 0.028
   b) 0.28
   c) 28
   d) 28000
   e) None of these. I think the answer is _______.

3. What number is one hundredth more than 2.468? _______

The percentage of students with the correct response was shown in Table 5:4. The scores increased after the teaching and again at the delayed post-test for question 2 but decreased for question 3 at the post-test yet increased for the eight students in the study at the delayed post-test.

Table 5:3
Percentage of Correct Responses to Items 1 and 10

Question	**Pretest** **N = 29**	**Post-test** **N = 8**	**Delayed Post-test** **N = 6**
1. a)	28%	75%	67%
b)	7%	13%	13%
10. a)	7%	13%	0%
b)	10%	13%	13%

Table 5:4
Percentage of Correct Responses to Items 2 and 3

Question	Pretest N = 29	Post-test N = 8	Delayed Post-test N = 6
2	34%	50%	67%
3	66%	38%	50%

The majority of students responded correctly to both questions. The students' responses indicated that they had less difficulty when dealing with specified place values of a given decimal number than with questions dealing with the interpretation of place value.

During the delayed interviews it was revealed why in some cases the 4 in the tenths place (question 3) was changed rather than the 6 in the hundredths place. Figure 5:2 showed that some students were using the place values for whole numbers starting from the extreme right of the decimal number. For these students the number in the tenths place represented the number in the hundredths place. Ryan displayed another difficulty. During the teaching experiment, when the worksheet questions asked that a certain place value be increased or decreased by one, the digit to the furthest right was always the one that was worked with regardless of its place value.

Scale Reading

The scores for the majority of questions dealing with scale reading increased after the teaching. The questions on scale reading were found in Figures 5:3 and 5:4. The scores for questions 4 and 5 (scale reading) were shown in Table 5:5. The students had no difficulty in reading a scale that was divided into tenths (item 4a and 4c); yet for the eight students in the study the scores went down on the delayed post-test. The scores for items 4g and 4h (scales subdivided into fifths) increased after the teaching and again at the delayed post-test. Very few students interpreted a fifth as two tenths. They used visual clues; two spaces after the whole number meant point 2. Visual clues were also used for items 4e and 4f (scales subdivided into hundredths); three spaces after 2 gave 2.3 and seven spaces after 2.1 gave 2.7. The results for item 4b and 4d (estimating a reading between marked calibrations) remained low throughout the study. The scores increased slightly in item 4b but decreased slightly in item 4d. Item 4i posed no difficulty as the scores increased on the delayed post-test.

The scores for question 5 showed a fluctuation in increasing and decreasing scores. The students were more confident with scale reading on a number line. Some of the students used fractions instead of decimals as the question asked and, as a result their answers were judged incorrect. This differed from question 4, where the decimal point was given which served as a hint to the students.

Fraction-to-Decimal Translation

Most students displayed difficulty when rewriting a fraction as a decimal. Questions 14 and 15 dealt with fraction-to-decimal translation. The questions were:

Figure 5:2
Place Value Interpretation of the Decimal Number 2.468

2 . 4 6 8

hundreds tens ones

Figure 5:3
Scale Reading

4. Read the following scales and write your answers in the boxes provided. Give all your answers as DECIMALS.

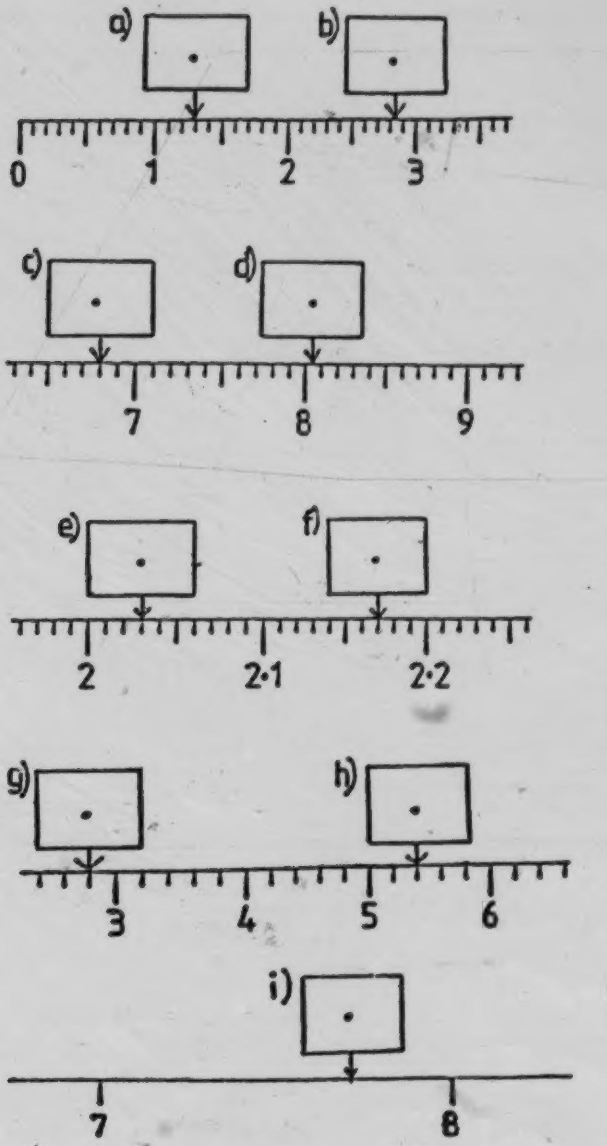
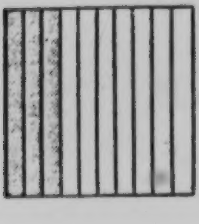

a)
b)
0
1
2
3
c)
d)
7
8
9
e)
f)
2
2·1
2·2
g)
h)
3
4
5
6
i)
7
8

Figure 5:4
Scale Reading

5. What decimal has been shaded in each of the following ?

1.



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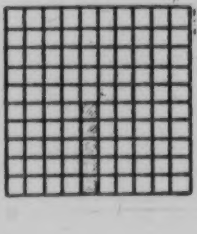
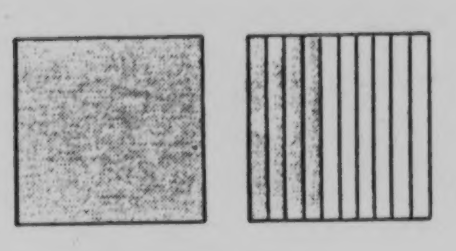
2.



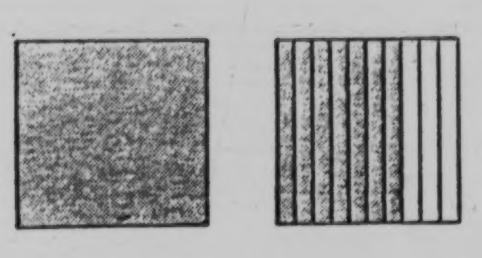
____

3.



____

4.



____

5.

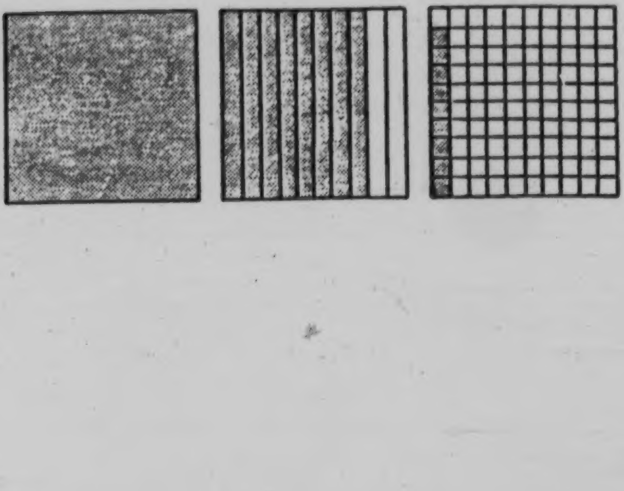


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Table 5:5
Percentage of Correct Responses to Items 4 and 5*

Question		Pretest N = 29	Post-test N = 8	Delayed Post-test N = 6
4	a	86%	88%	83%
	b	17%	13%	17%
	c	62%	88%	50%
	d	14%	38%	33%
	e	17%	38%	33%
	f	34%	50%	67%
	g	10%	25%	50%
	h	10%	25%	50%
	i	66%	63%	83%
5	a	79%		
	b	62%		
	c	21%		
	d	10%		
5	1		50%	33%
	2		25%	33%
	3		50%	33%
	4		50%	33%
	5		25%	33%

* Item 5 was not the same on the three tests.

14. Which decimal is equal to the fraction 3/5?

a) 0.35
b) 3.5
c) 0.6
d) 0.53
e) I don't know.

15. What are the decimal numbers for the following fractions?

a) 1/2 = _________
b) 1/5 = _________
c) 1 4/5 = _________
d) 2/3 = _________

The scores on these questions increased after the teaching and again at the delayed post-test. Table 5:6 outlined the percentage of correct responses for items 14 and 15. The majority of students did not consider the value of the fraction but used visual clues. At the post-test, 62% of the students responded with a syntactic translation for question 14 whereas at the delayed post-test, 50% of the students responded with a syntactic translation. Most students knew that 1/2 is 0.5, however, 17% of the students on the delayed post-test responded with a syntactic translation for the fraction 1/2. The responses included 1.2 and 0.12. The responses for the fractions 1/5 and 2/3 were varied. Seven students (32%) responded with a syntactic translation for the fraction 1/5 and 27% responded with a syntactic translation for the fraction 2/3.

During the teaching experiment it was noticed that the students did not think about the value of the fraction and tried to interpret it as a decimal. The students were concerned about finding a procedure that would produce the right answer. When they did not know the answer they wanted a procedure to use, not a thinking process.

Operations

The best performance of the students was displayed in the questions dealing with operations. These questions were:

7. Perform the following subtractions.

a) 7.81 - 3.5 = _________
b) 4.79 - 1.38 = _________
c) 43.20 - 0.16 = _________

Table 5:6
Percentage of Correct Responses to Items 14 and 15

Question		**Pretest** **N = 29**	**Post-test** **N = 8**	**Delayed Post-test** **N = 6**
14		10%	38%	50%
15	a	45%	63%	83%
	b	31%	50%	83%
	c	no	comparable	item
	d	17%	38%	50%

8. Perform the following multiplications.

a) 5.3 x 4 = _________
b) 8.4 x 2.1 = _________
c) 0.27 x 0.03 = _________

11. Perform the following additions.

a) 5.6 + 2.3 = _________
b) 6.3 + 3.42 = _________
c) 6.1 + .66 = _________

12. Perform the following divisions.

a) 24.6 ÷ 6 = _________
b) 0.135 ÷ 5 = _________
c) 4.014 ÷ 0.03 = _________

The percentage of correct responses for these items was summarized in Table 5:7. The scores on the addition items remained high, whereas the scores for the subtraction and multiplication questions increased after the teaching intervention and in most cases, again at the delayed post-test. The students seemed quite confident with the operation of addition and found the operation of division the most difficult.

The students were looking for a rule that would give them the correct answer. Questions such as; "Is this correct?", "Do I have the right answer?", "Show me how to do this?", suggested that the students were unsure of their procedures and had nothing else upon which to rely. Some students displayed difficulties in addition and subtraction by not lining up the decimal points and adding or subtracting corresponding place values.

Summary

The students taking part in the teaching experiment were mostly concerned about procedures that would give them the correct answer. The four areas of the teaching experiment have been summarized individually.

Place Value

The students were most comfortable when asked to work with a specified place value rather than with using the interpretation of place values to compare decimal numbers. For example, when asked to circle the decimal numbers that were smaller

Table 5:7
Percentage of Correct Responses to Items 7, 8, 11 and 12

Question		Pretest N = 29	Post-test N = 8	Delayed Post-test N = 6
7	a	34%	88%	100%
	b	83%	100%	100%
	c	79%	88%	83%
8	a	76%	88%	83%
	b	62%	75%	100%
	c	45%	75%	100%
11	a	93%	100%	100%
	b	86%	88%	100%
	c	76%	88%	100%
12	a	52%	75%	83%
	b	21%	13%	33%
	c	38%	25%	50%

than 0.75, one student circled the decimal numbers having only one decimal place. To this student, smaller meant less than two decimal places.

When they could refer to decimal numbers in terms of money, there was less difficulty with the problem. They enjoyed the worksheet "Fewest Number of Coins" (Appendix E), and could handle money efficiently.

Their performance at ordering decimal numbers improved when they were given concrete materials with which they could work. They were each given ten index cards with one decimal number per card and asked to arrange the cards from smallest to largest. These cards, although they were not a concrete decimal place value model, gave the students something to manipulate. They found it easier to think about place value with the cards than without. They would lay them down and move them around.

The exercises dealing with the abacus were enjoyed by the students; however, they found these exercises somewhat simplistic. There was no thinking about place value at all when doing these exercises. The last worksheet in Appendix E required the students to place one decimal point and three marks on the abacus to form as many different decimal numbers as possible. The students did not consider decimal numbers at all but mechanically put three marks and one decimal point on each abacus. When they were asked to compare their worksheet with worksheets from the rest of the group, they did not compare decimal numbers but they compared the placement of the marks and the decimal point.

The worksheets proved valuable as the scores on the questions dealing with place value did improve.

Scale Reading

Scale reading improved with the teaching experiment. After the teaching there was still difficulty with the reading of points between marked calibrations. Some students continued to use fractions and decimals together. When the arrow pointed to a reading of 3.05, students were writing 3.1/2 because it was halfway between 3.0 and 3.1.

Fraction-to-Decimal Translation

The students seemed to have more difficulty when comparing fractions and decimals to a given fraction than translating from one to the other. Scores on the items dealing with fraction-to-decimal translation did improve.

Operations

The majority of the group was quite confident with addition and subtraction but were unsure when to multiply or divide. They thought that a percentage of something meant a division. They had no notion of what to do with 84% of 236. They were unable to say whether the answer would be more or less than 236.

Two students out of the eight were able to find the pattern in "Completing the Pyramid" (Appendix H). They were not receptive to probing questions from the teacher/researcher to help them discover the pattern.

The students displayed a good performance on the questions dealing with operations and the scores did increase after the teaching experiment.

Teaching Experiment

It was felt by the researcher that the teaching experiment was a success. The scores did increase after the teaching and most of the scores increased again at the delayed post-test. Observations made during the teaching experiment suggested that these particular students were mainly interested in procedures that would produce a correct answer. Working in groups on worksheets also seemed successful. As the experiment progressed, the groups became more at ease with the other members of the group and with the researcher. They shared their answers and observations; however, they did not share their thinking process. Only when probed did they share their thinking process.

The student worksheets and responses are discussed in detail in Chapter VI. This discussion led to the placement of the students on the Onion Model of Decimal Number Learning.

CHAPTER VI

Analysis of Students' Worksheets

Introduction

This chapter will discuss the students' responses to the worksheets used in the teaching experiment. The worksheet responses and the interaction between the members of each group and the interaction among the students in the study and the teacher/researcher led the teacher/researcher to formulate certain perceptions regarding the decimal number knowledge of these students. Their mathematical background, their way of thinking and their capability to reconstruct decimal number knowledge were also examined.

The analysis of individual students that follows summarized the student responses about the activities on the worksheets, how the activities were perceived by each student and the cognitive interactions. This data allowed each student to be placed on the Onion Model of Decimal Number Learning as perceived by the teacher/researcher.

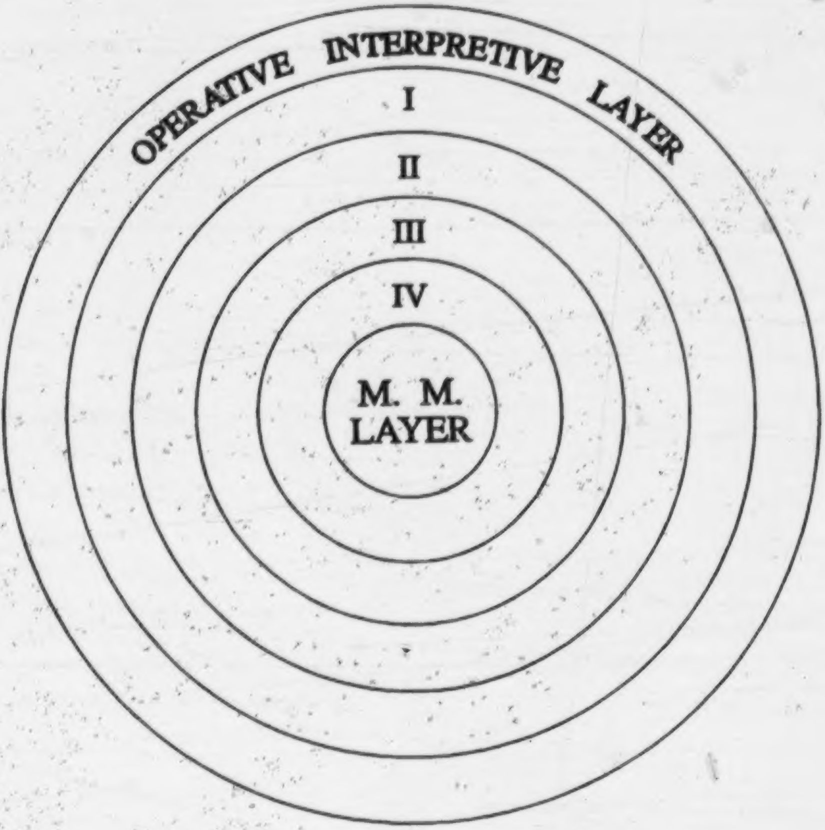
The students taking part in the teaching experiment were all placed at the procedural layer of the Onion Model after the data were examined and analyzed. Although the students were placed at the same layer, differences did exist among the students. These differences existed in the degree of procedural knowledge, attitude, motivation, confidence and existing personal emotions. For these reasons, the procedural layer was subdivided into levels. The levels were to accommodate these differences and to place each student closer to or farther away from the abstract layer.

The different levels of the procedural layer (Figure 6:1) could be characterized as follows:

Level I

- poor procedural knowledge
- could specify a place value position
- could perform the operations of addition, subtraction and multiplication
- knew the decimal equivalent for the fraction 1/2
- did not know what action to take when faced with a given situation
- procedural knowledge did not allow for efficient functioning within the decimal number system
- poor procedural knowledge led to incorrect responses
- low motivation and showed only some interest in the procedures
- was not likely to proceed beyond the procedural layer.

Figure 6:1
The Four Levels of the Procedural Layer


OPERATIVE INTERPRETIVE LAYER
I
II
III
IV
M. M.
LAYER

Level II

- good procedural knowledge
- could specify a place value position
- could perform the operations of addition, subtraction and multiplication
- could read scales subdivided into tenths
- could order decimals
- knew the decimal equivalent for the fraction 1/2
- could take action when a situation presented itself
- procedural knowledge allowed for a somewhat successful functioning within the decimal number system
- showed interest in the procedures
- showed no interest in knowing why the procedure worked but showed an interest in knowing how a procedure worked.

Level III

- very good procedural knowledge
- could specify a place value position
- could perform the operations of addition, subtraction, multiplication and division
- could read scales subdivided in units other than tenths
- could order and compare decimals
- could translate fractions to decimals
- could take effective action when faced with a situation
- procedural knowledge allowed for an efficient functioning within the decimal number system; few errors
- showed interest beyond the procedures; did not want to know how the procedure works but could be led to deeper insights when questioned
- could likely peel the procedural layer.

Level IV

- excellent procedural knowledge
- could specify a place value position
- could perform the operations of addition, subtraction, multiplication and division
- could read scales subdivided in units other than tenths
- order and compare decimals
- could translate fractions to decimals
- could specify a correct operation to a given written situation
- could take effective action with nearly no errors when faced with a situation
- procedural knowledge allowed for an efficient functioning within the decimal number system
- showed interest beyond the procedures
- did not want to know why a procedure worked but revealed deeper insights when

questioned
- could likely peel the procedural growth layer
- could possibly attain the abstract layer.

These levels helped to place the students on the Onion Model based upon their performance in the study.

Analysis of Individual Students

One purpose of the study was to gain insights into the reconstruction of decimal knowledge in students having a base of procedural knowledge. Observation and questioning were used to find out what techniques and processes were used by the students while they were engaged in completing the decimal number worksheets.

The content of the worksheets dealt with four aspects of decimal number knowledge: place value, scale reading, fraction-to-decimal translation and operations as described in Chapter V. The results of each of the eight students who took part in the teaching experiment were analyzed individually using these four content areas of decimal number knowledge. The names of the students have been changed to grant anonymity.

PRESENTATION OF TYLER'S WORK

Tyler completed nearly all the worksheets (Figure 6:2). The following sections summarized his performance during the teaching experiment.

Place Value

Tyler demonstrated that he could take effective action when faced with situations involving place value. He correctly identified a specific place value position when comparing decimal numbers as demonstrated with worksheets E-4 and 5 (Figure 6:3).

Tyler did display some inconsistencies on worksheet E-6 (Figure 6:4). Item one required him to write the number that is one tenth larger than a given number. Item 1b showed that he changed a correct answer (2.0) to an incorrect answer (1.10). It is difficult to say why he changed 2.0 to 1.10. He may have looked at Ryan's paper (one of his groupmates), decided that Ryan's answer (1.10) looked correct and used it. It can be noted that in question 1 Tyler changed the last digit in each item. Had Tyler left 2.0 as a response for item 1b, that procedure would have given an incorrect response for item 1d only.

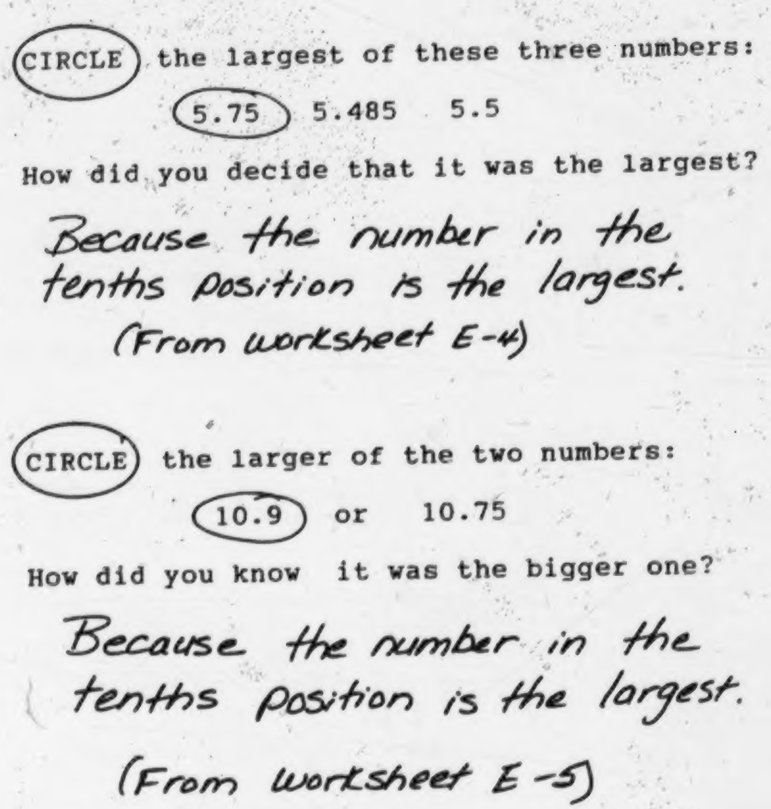
Tyler correctly changed all the items in question 2 to a number that was one hundredth greater than the given number. Here he correctly changed the nine

Figure 6:2
Tyler's Worksheets

Place Value		Scale Reading		Fraction-to-Decimal Translation		Operations	
E-1		F-1		G-1	✓	H-1	✓
E-2		F-2		G-2	✓	H-2	✓
E-3	✓	F-3		G-3	✓	H-3	✓
E-4	✓	F-4		G-4	✓	H-4	✓
E-5	✓					H-5	✓
E-6	✓					H-6	✓
E-7	✓						
E-8	✓						
E-9	✓						
E-10	✓						
E-11	✓						

✓ Completed   A Attempted   C Copied

Figure 6:3
Tyler's Place Value Examples



CIRCLE the largest of these three numbers:

[✓ 5.75] 5.485 5.5

How did you decide that it was the largest?

Because the number in the tenths position is the largest.

(From worksheet E-4)

CIRCLE the larger of the two numbers:

[✓ 10.9] or 10.75

How did you know it was the bigger one?

Because the number in the tenths position is the largest.

(From worksheet E-5)

Figure 6:4
Tyler's Worksheet E-6

1. Write the number that is one tenth greater.

a) 2.5 2.6
b) 1.9 ~~2.0~~ 1.10
c) 25.7 25. 8
d) 0.68 0.69
e) 125.2 125 . 3
f) 0.7 0.8

2. Write the number that is one hundredth greater.

a) 8.75 8. 76
b) 0.93 0.94
c) 1.085 1.095
d) 3.346 3.356
e) 17.591 17.601
f) 0.083 0.093

3. Write the number that is one thousandth less.

a) 5.238 5.239
b) 6.088 6.089
c) 0.13 0.14
d) 4.959 4.9510
e) 6.781 6.782
f) 15.326 15.326

hundredths to ten hundredths and carried this over to the tenths.

Tyler misread the directions for question 3 and performed additions rather than subtractions. Here again, he increased the last digit of each decimal number by one.

When utilizing the worksheets dealing with the abacus, Tyler simplified the expanded form of the decimal number before representing the decimal number on the abacus (Figure 6:5). He was able to represent questions 1 to 4 without further simplification but he was unable to do the same for questions 5 to 9. After simplification, he was better able to distinguish the tens, units and tenths; he was therefore better able to represent the decimal number on the abacus.

Scale Reading

Tyler was absent when the worksheets on scale reading were completed and he did not want to take them up the next day.

Fraction-to-Decimal Translation

Tyler was able to translate fractions into decimals and decimals into fractions quite easily. The cards (Figure 6:6) proved to be a success. Tyler pondered the cards carefully before making equivalent triplets, then he correctly transferred this information onto worksheet G-1 (Figure 6:7). He did ask for Julie's help (one of his groupmates) a number of times.

Difficulty arose when the fractions, decimals and percents were to be compared to a given fraction (Figure 6:8). The difficulty seemed to be that Tyler was faced with too much at once. The rectangle containing the fractions, decimals and percents seemed to overwhelm him. Had there been fewer numbers in the rectangle it seemed that the task would have appeared easier.

The task of changing a percent to a decimal or a decimal to a percent was a mechanical process for Tyler. He knew the procedure of moving the decimal point two places to the left or the right and applied this procedure correctly.

Operations

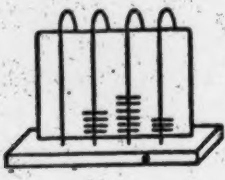
Tyler displayed good procedural skills when performing additions, subtractions, multiplications and divisions. He was able to distinguish which operation was to be used in the written problems.

Worksheet H-4 (Complete the Pyramid) was quickly finished by Tyler. He saw the pattern rapidly and was then asked by the other members of his group about the solution. By using triangles he was able to show the members of his group how two

Figure 6:5
Tyler's Worksheet E-7

DECIMALS AND THE ABACUS

46.3 can be shown on an abacus

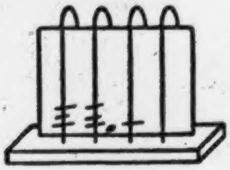


The number 46.3 is written below in expanded form.

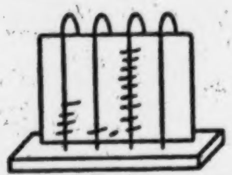
40 + 6 + 0.3

= 4 tens + 6 ones + 3 tenths
= (4x10) + (6x1) + (3x0.1)
= (4x10) + (6x1) + (3x1/10)

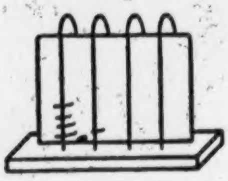
A.
Show each of the following expanded forms on the abacus.



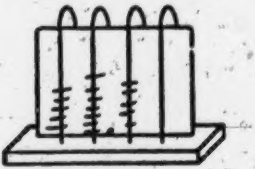
1. 30 + 3 + 0.1



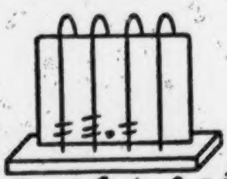
2. 40 + 1 + 0.9



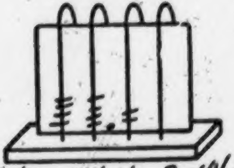
3. 4 + 0.1



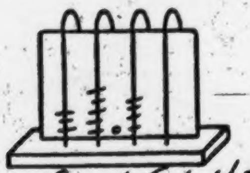
4. 50 + 7 + 0.5



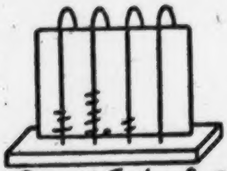
20 + 3 + .2 = 23.2
5. (5 x 4) + (3 x 1) + (2 x 0.1)



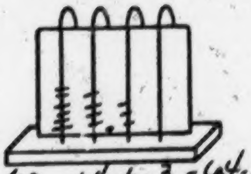
40 + 4 + .2 = 44.2
6. (4 x 10) + (4 x 1) + (2 x 0.1)



30 + 5 + .4 = 35.4
7. (3 x 10) + (5 x 1) + (4 x 0.1)



30 + 5 + .2 = 35.2
8. (3 x 10) + (5 x 1) + (2 x [illegible])



60 + 4 + .3 = 64.3
9. (6 x 10) + (4 x 1) + (3 x [illegible])

Figure 6:6
Finding Equivalent Triplets

0.90	0.8	$\frac{1}{3}$	75%	10%
25%	$\frac{3}{5}$	$\frac{90}{100}$	100%	$\frac{4}{5}$
$\frac{1}{4}$	0.20	1.0	$\frac{5}{5}$	20%
0.1	90%	0.5	0.25	33.3%
$0.3\bar{3}$	0.6	0.75	80%	$\frac{1}{2}$
$\frac{3}{4}$	$\frac{1}{5}$	$\frac{1}{10}$	50%	60%

Figure 6:7
Tyler's Worksheet G-1

FRACTIONS

1/2 3/5 1/10 1/4 1/5

90/100 5/5 1/3 4/5 3/4

DECIMALS

0.1 1.0 0.5 0.90 0.6

0.25 0.20 0.75 0.8 0.33

PERCENTS

90% 50% 60% 100% 20%

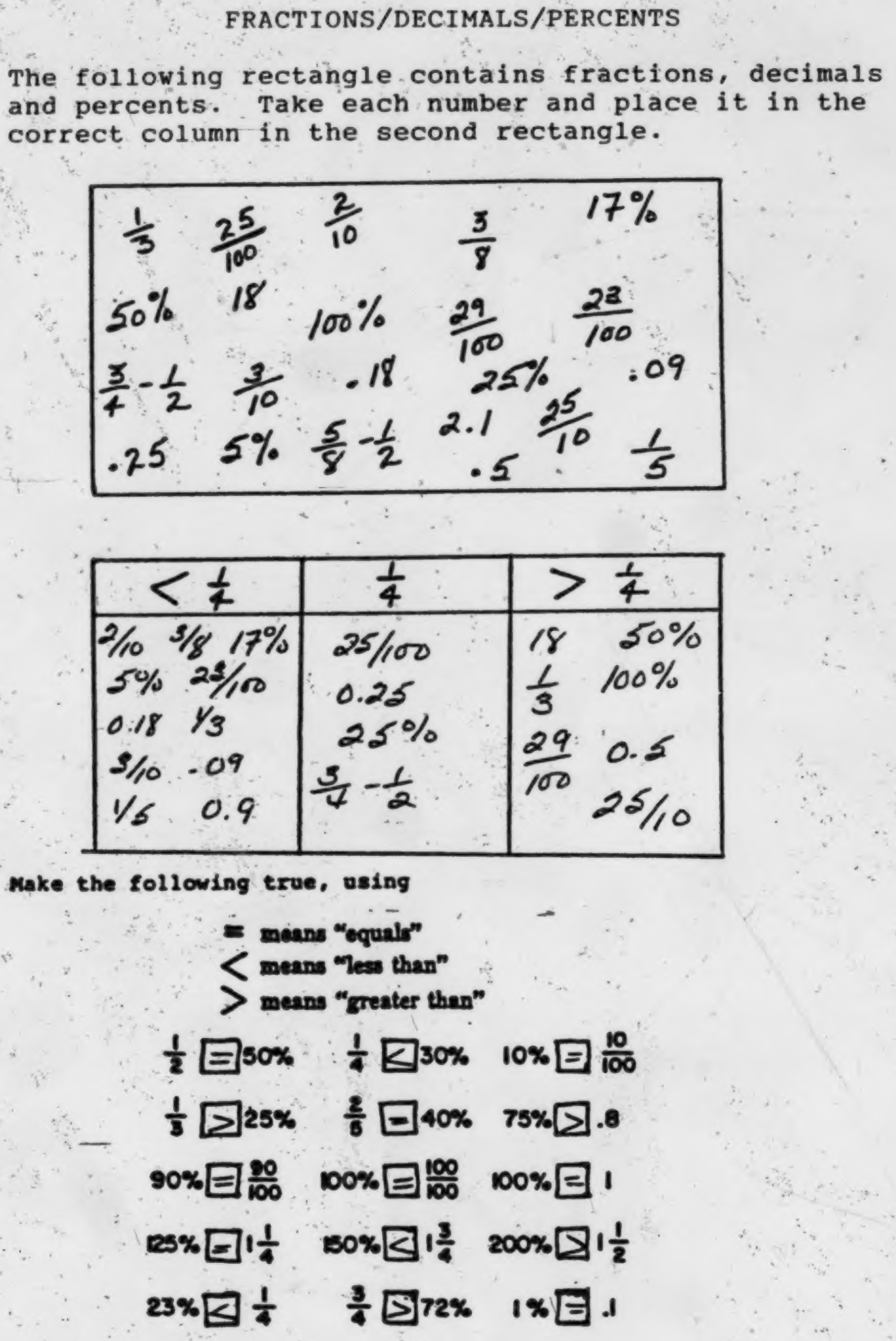
33.3% 10% 25% 80% 75%

INSTRUCTIONS:

For each of the above fractions there is an equivalent decimal and percent. Fill the chart below with the fractions in the list and the decimal and percent that is equal to each fraction.

FRACTIONS	DECIMALS	PERCENTS
$\frac{1}{10}$	0.1	10%
$\frac{1}{5}$	0.20	20%
$\frac{1}{4}$	0.25	25%
$\frac{1}{3}$	0.33	3.33%
$\frac{1}{2}$	0.5	50%
$\frac{3}{5}$	0.6	60%
$\frac{3}{4}$	0.75	75%
$\frac{4}{5}$	0.80	80%
$\frac{90}{100}$	0.90	90%
$\frac{5}{5}$	1.0	100%

Figure 6:8
Tyler's Worksheet G-4



FRACTIONS/DECIMALS/PERCENTS

The following rectangle contains fractions, decimals and percents. Take each number and place it in the correct column in the second rectangle.

$\frac{1}{3}$ $\frac{25}{100}$ $\frac{2}{10}$ $\frac{3}{8}$ 17%
50% 18 100% $\frac{29}{100}$ $\frac{23}{100}$
$\frac{3}{4}-\frac{1}{2}$ $\frac{3}{10}$ .18 25% :09
.25 5% $\frac{5}{8}-\frac{1}{2}$ 2.1 $\frac{25}{10}$ .5 $\frac{1}{5}$

$< \frac{1}{4}$	$\frac{1}{4}$	$> \frac{1}{4}$
2/10 3/8 17% 5% 23/100 0.18 1/3 3/10 .09 1/5 0.9	25/100 0.25 25% $\frac{3}{4}-\frac{1}{2}$	18 50% $\frac{1}{3}$ 100% $\frac{29}{100}$ 0.5 25/10

Make the following true, using

= means "equals"
< means "less than"
> means "greater than"

$\frac{1}{2}\ \boxed{=}\ 50\%$ $\frac{1}{4}\ \boxed{<}\ 30\%$ $10\%\ \boxed{=}\ \frac{10}{100}$

$\frac{1}{3}\ \boxed{>}\ 25\%$ $\frac{2}{5}\ \boxed{=}\ 40\%$ $75\%\ \boxed{>}\ .8$

$90\%\ \boxed{=}\ \frac{90}{100}$ $100\%\ \boxed{=}\ \frac{100}{100}$ $100\%\ \boxed{=}\ 1$

$125\%\ \boxed{=}\ 1\frac{1}{4}$ $150\%\ \boxed{<}\ 1\frac{3}{4}$ $200\%\ \boxed{>}\ 1\frac{1}{2}$

$23\%\ \boxed{<}\ \frac{1}{4}$ $\frac{3}{4}\ \boxed{>}\ 72\%$ $1\%\ \boxed{=}\ .1$

adjacent numbers were added to make a third number directly below the two adjacent ones (Figure 6:9). He felt pleased that he had discovered the pattern and was now able to help his groupmates.

Interpretation of Tyler's Work

Tyler was a pleasant young man with a calm disposition. He never became frustrated, impatient or upset when he came across a problem for which he didnot know the answer. When he was uncertain as to how to proceed he simply asked his groupmates for help and among them they would try to recall a procedure that would help them in this situation.

Tyler displayed good procedural knowledge that allowed him to function efficiently within the decimal number system. His procedural knowledge permitted him to take effective action when faced with a decimal problem-solving situation. He could perform all operations and was able to distinguish which operation to use in a given situation.

Although Tyler did not show a love for mathematics, he displayed an openness that the other students did not possess. This openness allowed him to examine and ponder situations before arriving at a conclusion. The following question posed no problem for Tyler:

> The gunwale of a boat is 2.5 m above water. If the tide rises 1m, how much of the gunwale of the boat is now above water?

After pondering the question he replied that nothing would change, whereas the other members of the group completed a subtraction.

Tyler could be placed at level III of the procedural layer. He did not display any interest in anything other than the procedures, yet his listening skills and his observation skills suggested that he could go beyond the procedural layer.

Tyler wrote the pretest and the post-test but he did not write the delayed post-test as he quit school before the delayed post-test was administered. His post-test score was 54% which was higher than the pretest and the post-test mean. However, there is no data that showed whether or not an increase would have been retained to the delayed post-test.

PRESENTATION OF RYAN'S WORK

Ryan completed nearly all the worksheets (Figure 6:10). His performance was summarized in the following sections.

Figure 6:9
Tyler's Worksheet H-4

COMPLETE THE PYRAMID

Study the following pyramid to see how the numbers are obtained.

Complete the pyramid by supplying the missing numbers.

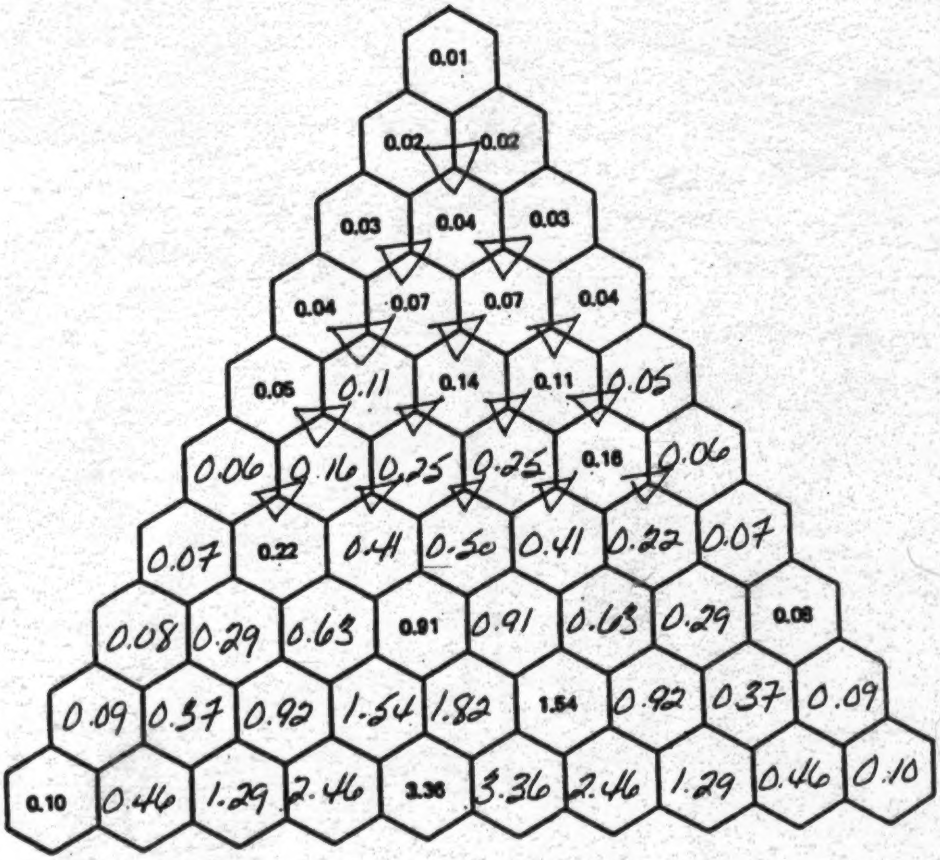

0.01
0.02 0.02
0.03 0.04 0.03
0.04 0.07 0.07 0.04
0.05 0.11 0.14 0.11 0.05
0.06 0.16 0.25 0.25 0.16 0.06
0.07 0.22 0.41 0.50 0.41 0.22 0.07
0.08 0.29 0.63 0.91 0.91 0.63 0.29 0.08
0.09 0.37 0.92 1.54 1.82 1.54 0.92 0.37 0.09
0.10 0.46 1.29 2.46 3.36 3.36 2.46 1.29 0.46 0.10

Figure 6:10
Ryan's Worksheets

Place Value		Scale Reading		Fraction-to-Decimal Translation		Operations	
E-1		F-1		G-1	C	H-1	✓
E-2		F-2		G-2	C	H-2	✓
E-3	✓	F-3		G-3	C	H-3	✓
E-4	✓	F-4		G-4	A	H-4	✓
E-5	✓					H-5	✓
E-6	✓					H-6	✓
E-7	✓						
E-8	✓						
E-9	✓						
E-10	✓						
E-11	✓						

/ Completed A Attempted C Copied

Place Value

Ryan completed nine of the eleven place value worksheets, with Worksheets E-3, 4, and 5 being fairly well done. Upon closer examination, it was found that these worksheets showed some inconsistencies and displayed Ryan's difficulty when ordering decimals. The responses displayed in Figure 6:11:A were Ryan's responses to the given question. After completing the worksheet he checked his responses with Julie's (a groupmate). The answers were not the same and he then made a line through his responses and copied Julie's responses (Figure 6:11:B). Julie's responses were not questioned by Ryan as he seemed to think that she was a better student at math than he was.

Ryan responded correctly to the questions where he was asked to compare two decimal numbers and explain why his response was correct (Figure 6:12). His expression "because the number after the decimal is bigger" suggested that he was referring to the tenths place which would give a correct response although he was unable to say this specifically. However, Figure 6:13 showed that the rule was applied in a different manner. Figure 6:13:A and 6:13:C showed a correct response whereas B and D did not. Item A and C gave a decimal number which contained one digit after the decimal point. The response was based on a decimal number that had only one digit after the decimal point. If that decimal number had a larger digit after the decimal point it was judged correct and in these cases, it did give the correct response. Item B and D were decimal numbers which contained two digits after the decimal point. However, as Ryan based his responses on decimal numbers that had only one digit after the decimal point, his responses contained only one digit after the decimal point. His circled responses reflected this and were correct but he failed to recognize other responses which were also correct.

Ryan's major concern was to fill in the spaces on the worksheet. He was not worried whether the answer was correct or not. If he was unable to think of a procedure to use, he would copy the responses from another member of his group or would ask the teacher/researcher for the proper procedure. Ryan requested assistance with worksheet E-6 (Figure 6:14) from another student in his group. He asked, "How do you do this?" and Julie replied, "Add one to the last number." Ryan did as he was told except for item 1c which he did not change at all. Adding one to the last number gave the correct response to items 1. a, e, and f but this did not display knowledge of the tenths place. His responses for items 2 and 3 also showed a lack of knowledge about the place value system. He did exactly as he had been told by Julie. He added one to all the last digits even though question 3 asked for subtraction.

Ryan found the solution to each decimal number in expanded form before placing its representation on the abacus (Figure 6:15). Items 1 to 4 were easily done without simplifying the expanded form. Items 5 to 9 were simplified first. The simplified form made it easier for him to represent the tens, units, and tenths on the abacus.

Figure 6:11
Ryan's Place Value Examples

Write the following numbers in ascending order (SMALLEST first).

3.87   3.9   4.07   4.5   3.49   3.785   3.6

(From Worksheet E-4)

(A) ~~6 7 1 3 2 5 4~~

(B) 4 5 6 7 1 3 2

Figure 6:12
Ryan's Place Value Examples

CIRCLE the largest of these three numbers:

[✓ 5.75] 5.485 5.5

How did you decide that it was the largest?

(From worksheet E-4)

Because the number is bigger after the decimal.

CIRCLE the larger of the two numbers:

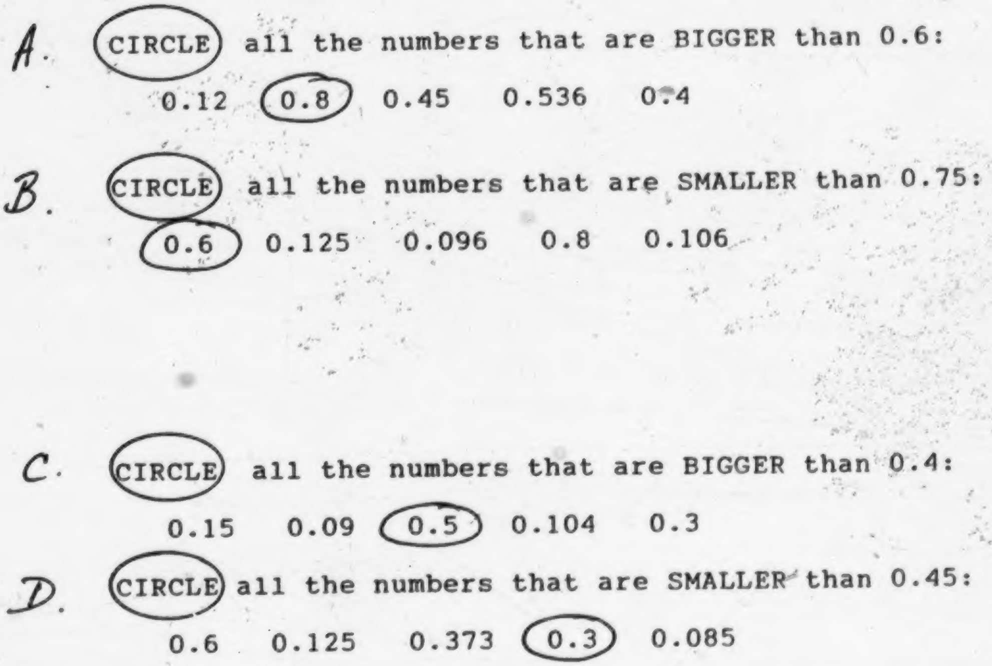
[✓ 10.9] or 10.75

How did you know it was the bigger one?

(From worksheet E-5)

Because the number after the decimal is bigger.

Figure 6:13
Ryan's Place Value Examples



A. CIRCLE all the numbers that are BIGGER than 0.6:

0.12 [✓ 0.8] 0.45 0.536 0.4

B. CIRCLE all the numbers that are SMALLER than 0.75:

[✓ 0.6] 0.125 0.096 0.8 0.106

C. CIRCLE all the numbers that are BIGGER than 0.4:

0.15 0.09 [✓ 0.5] 0.104 0.3

D. CIRCLE all the numbers that are SMALLER than 0.45:

0.6 0.125 0.373 [✓ 0.3] 0.085

Figure 6:14
Ryan's Worksheet E-6

1. Write the number that is one tenth greater.

a) 2.5 2.6
b) 1.9 1.10
c) 25.7 25.7
d) 0.68 0.69
e) 125.2 125. 3
f) 0.7 0.8

2. Write the number that is one hundredth greater.

a) 8.75 8.76
b) 0.93 0.93
c) 1.085 1.086
d) 3.346 3356
e) 17.591 17. 590
f) 0.083 0.093

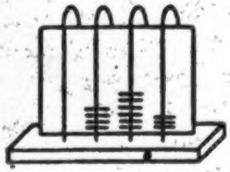
3. Write the number that is one thousandth less.

a) 5.238 5. 239
b) 6.088 6.089
c) 0.13 0.14
d) 4.959 4.9510
e) 6.781 6.782
f) 15.326 15. 326

Figure 6:15
Ryan's Worksheet E-7

DECIMALS AND THE ABACUS

46.3 can be shown on an abacus

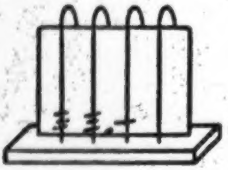


The number 46.3 is written below in expanded form.

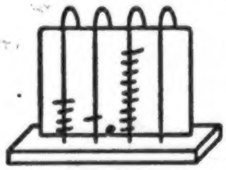
40 + 6 + 0.3

= 4 tens + 6 ones + 3 tenths
= (4x10) + (6x1) + (3x0.1)
= (4x10) + (6x1) + (3x1/10)

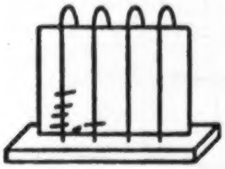
A.
Show each of the following expanded forms on the abacus.



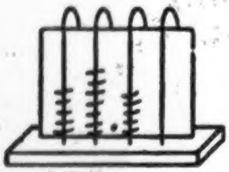
1. 30 + 3 + 0.1



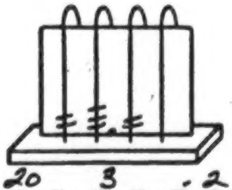
2. 40 + 1 + 0.9



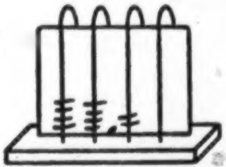
3. 4 + 0.1



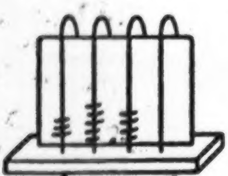
4. 50 + 7 + 0.5



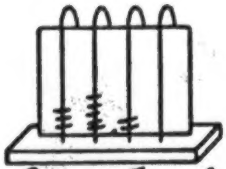
20 3 .2
5. (5 x 4) + (3 x 1) + (2 x 0.1)



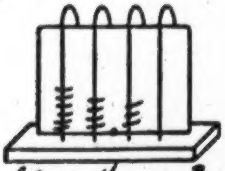
40 4 .2
6. (4 x 10) + (4 x 1) + (2 x 0.1)



30 5 .4
7. (3 x 10) + (5 x 1) + (4 x 0.1)



30 5 .2
8. (3 x 10) + (5 x 1) + (2 x [illegible])



60 4 .3
9. (6 x 10) + (4 x 1) + (3 x [illegible])

Worksheet E-11 required that the students place one decimal point and three marks on each abacus to form as many different representations of decimal numbers as possible. When Ryan had completed as many as he could (Figure 6:16) he asked, "What other ones are there?" He was then asked to write down the decimal numbers that he had completed. He wrote these decimal numbers directly under the abacus. He was then asked to consider other possibilities for the various place values but did not respond well to that idea and did not complete the page. He wanted to know what the numbers were and then he put them on the abacus. When he was asked to think about his answer, he decided that it would be easier not to complete the page. The possibility that he had all the answers did not enter his mind.

Ryan did not respond to questions dealing with a certain digit within the decimal number in terms of place-value position. His response always dealt with a number of marks on the abacus. The tenths place only meant a certain number of marks on the first rung of the abacus after the decimal point. Representing decimal numbers on the abacus became a mechanical process whereby a number of marks were placed on various rungs of the abacus with a decimal point inserted somewhere.

Scale Reading

Ryan was absent when the worksheets on scale reading were done. He did not take them up the following day.

Fraction-to-Decimal Translation

Ryan did not know how to compare fractions, decimals and percents and was content to let the other students in the group complete the worksheets from which he copied their answers. He made an effort to help his group with worksheet G-4 (Figure 6:17). He was not sure what the symbols < and > were and asked the teacher/researcher. This was almost his only effort to help the group.

Ryan was interested in the cards (Figure 6:6) requiring him to find equivalent triplets. He liked to hold the cards in his hand and shuffle them as in a game. The physical aspect of the cards seemed to be a motivation and he worked with these with interest. He did, however, seem to be working from memory. When he came across a card, whether fraction, decimal or percent, with which he was not familiar, he would ask Julie, "What's this one? Where does this one go?"

Operations

Ryan displayed good procedural knowledge concerning the addition and subtraction of decimal numbers. Multiplication and division were also performed well when Ryan knew which procedure to perform. The difficulty arose when he had to

Figure 6:16
Ryan's Worksheet E-11

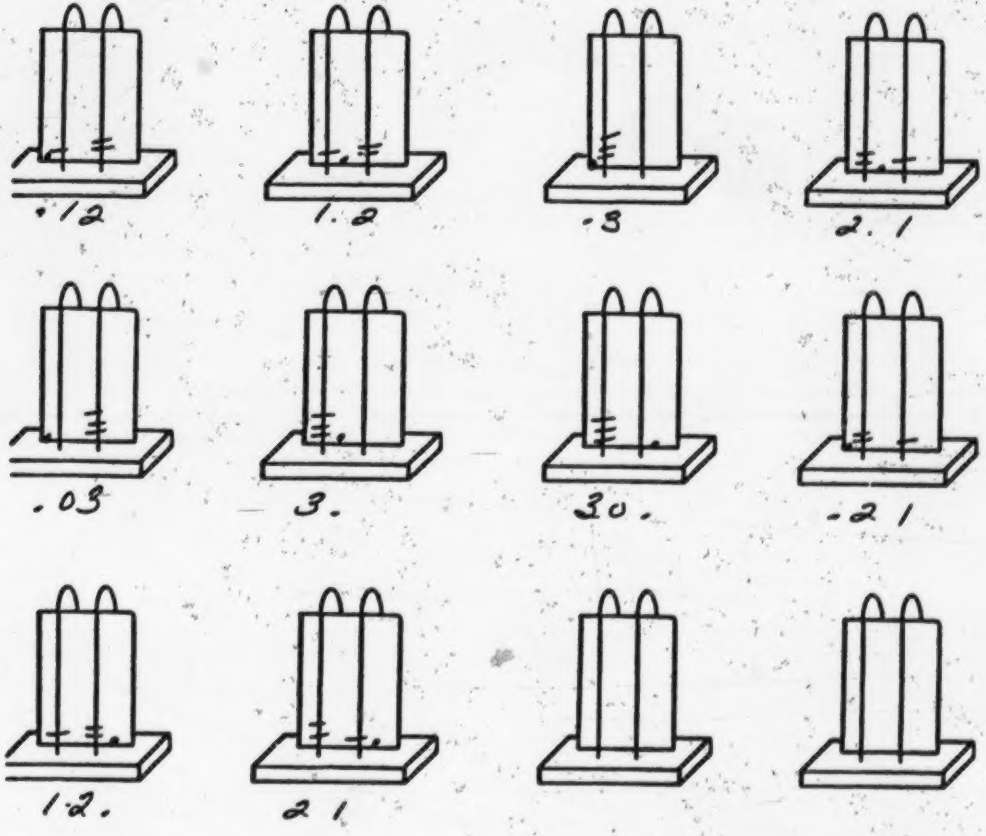
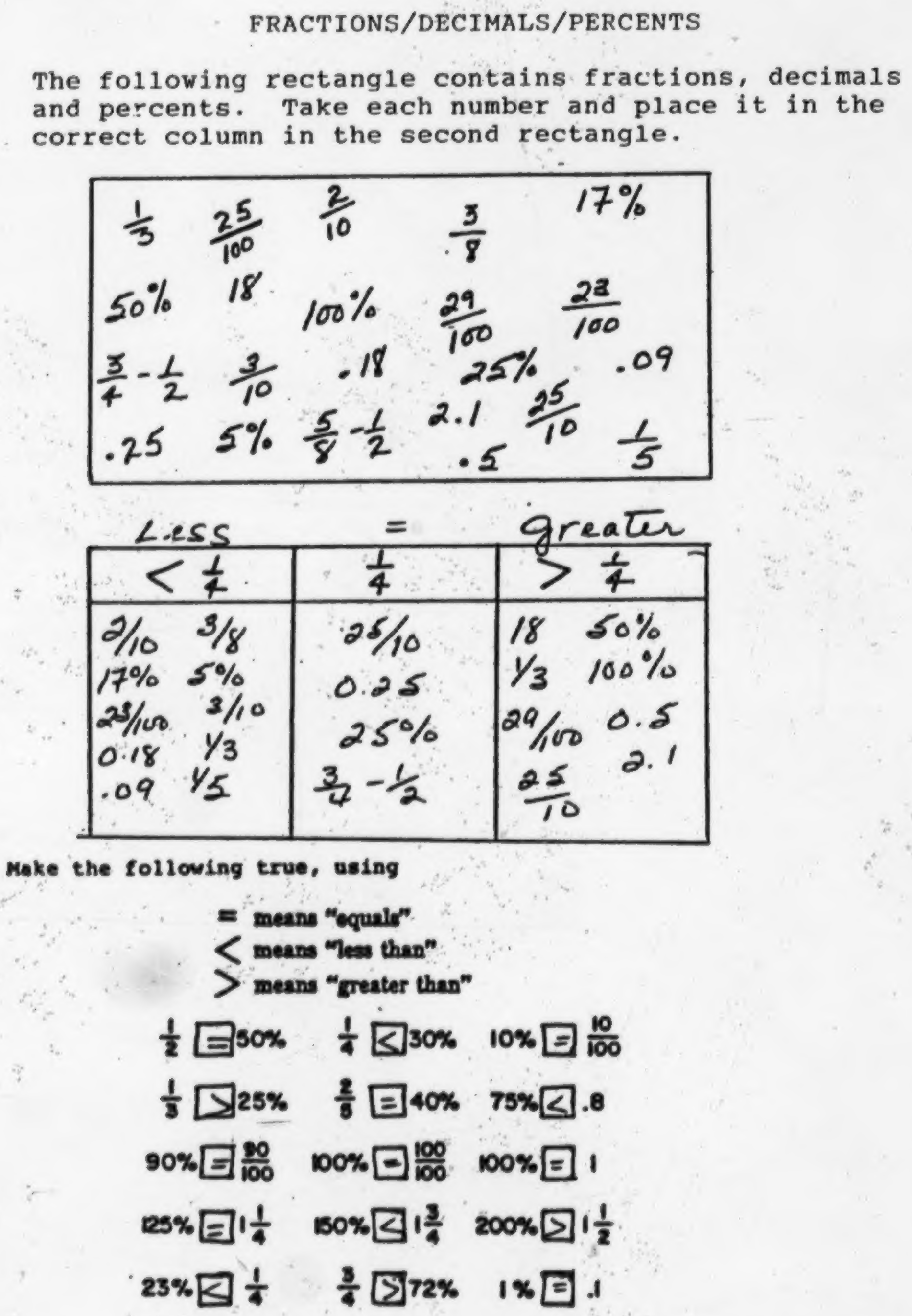

.12
1.2
.3
2.1
.03
3.
30.
.21
12.
21

Figure 6:17
Ryan's Worksheet G-4



FRACTIONS/DECIMALS/PERCENTS

The following rectangle contains fractions, decimals and percents. Take each number and place it in the correct column in the second rectangle.

$\frac{1}{3}$ $\frac{25}{100}$ $\frac{2}{10}$ $\frac{3}{8}$ 17%
50% 18 100% $\frac{29}{100}$ $\frac{23}{100}$
$\frac{3}{4}-\frac{1}{2}$ $\frac{3}{10}$ .18 25% .09
.25 5% $\frac{5}{8}-\frac{1}{2}$ 2.1 $\frac{25}{10}$ .5 $\frac{1}{5}$

Less = greater

$< \frac{1}{4}$	$\frac{1}{4}$	$> \frac{1}{4}$
2/10 3/8	25/10	18 50%
17% 5%	0.25	1/3 100%
23/100 3/10	25%	29/100 0.5
0.18 1/3	$\frac{3}{4}-\frac{1}{2}$	2.1
.09 1/5		$\frac{25}{10}$

Make the following true, using

= means "equals"
< means "less than"
> means "greater than"

$\frac{1}{2}\ \boxed{=}\ 50\%$ $\frac{1}{4}\ \boxed{<}\ 30\%$ $10\%\ \boxed{=}\ \frac{10}{100}$

$\frac{1}{3}\ \boxed{>}\ 25\%$ $\frac{2}{5}\ \boxed{=}\ 40\%$ $75\%\ \boxed{<}\ .8$

$90\%\ \boxed{=}\ \frac{90}{100}$ $100\%\ \boxed{=}\ \frac{100}{100}$ $100\%\ \boxed{=}\ 1$

$125\%\ \boxed{=}\ 1\frac{1}{4}$ $150\%\ \boxed{<}\ 1\frac{3}{4}$ $200\%\ \boxed{>}\ 1\frac{1}{2}$

$23\%\ \boxed{<}\ \frac{1}{4}$ $\frac{3}{4}\ \boxed{>}\ 72\%$ $1\%\ \boxed{=}\ .1$

decide whether a written problem was asking for multiplication or division. Rather than discuss the problem with his group members and try to arrive at a decision, he would ask them or the teacher/researcher whether it was a multiplication or a division question. To further hinder him, Ryan was unable to tell if the answer should be bigger or smaller than the number in the question. The following question appeared on worksheet H-6:

> A football stadium has seating for 68 300 people. If 84% of the seats are occupied, how many people are at the football game?

The teacher/researcher asked him if the answer should be more or less than 68 300. He did not know. The term "84% of" had no meaning for him. Rather than venture a guess at the final answer, he guessed an operation and said, "I think it's a multiply so the answer would be more". Ryan's procedural knowledge of the multiplication and division of decimal numbers was the same rule he used with whole numbers, multiplication makes bigger and division makes smaller.

A question dealing with perimeter arose in worksheet H-3. The group pondered over the word perimeter, but did not know the meaning of the word and asked the teacher/researcher for an explanation. Before the teacher/researcher responded Ryan shouted, "I know. It means all the way around." The group agreed with Ryan and answered the question without help from the teacher/researcher. This was the only time during the teaching experiment that Ryan gave a response and felt confident that his response was correct.

Ryan was unable to find the pattern in the Pyramid (Figure 6:18). He observed that the columns to the extreme left and extreme right should be identical and that the numbers increased from one to ten. He made no distinction between tenths and hundredths and he filled in the other spaces with responses from his group members.

Interpretation of Ryan's Work

Ryan liked the teaching experiment because it allowed him the opportunity to socialize with the members of his group. He stayed with the same group throughout the study and relied on his group members a lot, mostly Julie. He did volunteer a response now and then.

Ryan's procedural knowledge was limited and the action he took when faced with a situation involving decimal numbers was not always successful. His procedural knowledge allowed him to function at a minimal efficiency level and he was happy with this.

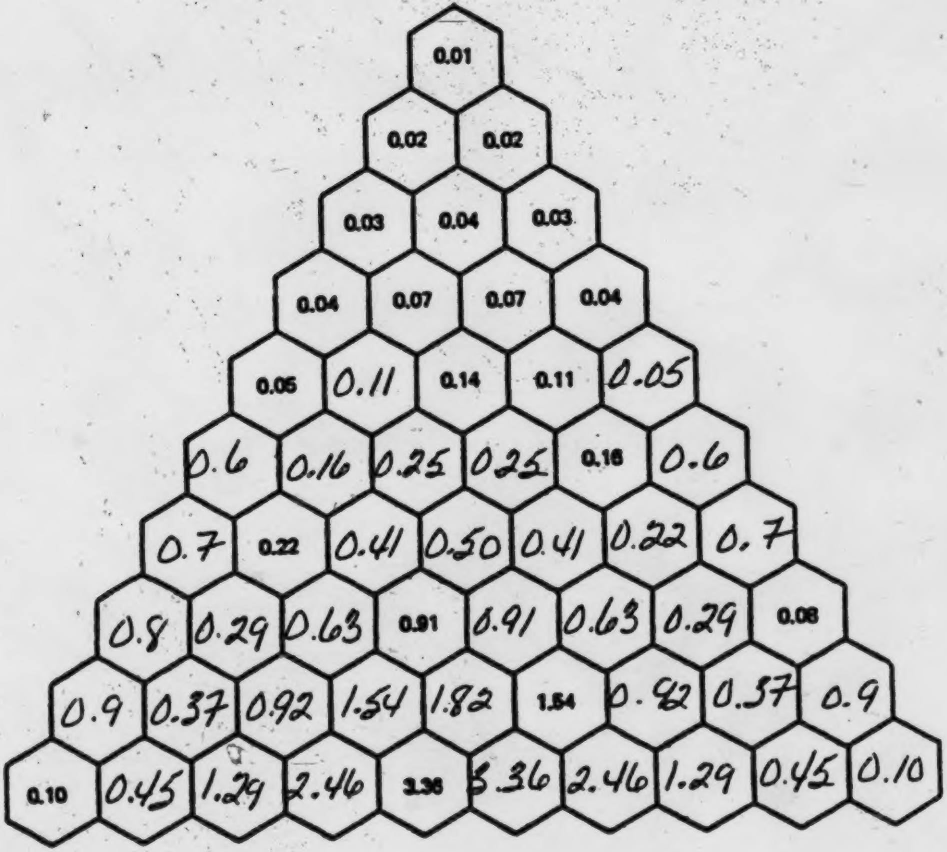
Ryan's lack of motivation, lack of interest in school and mathematics and his social standards permitted the teacher/ researcher to place Ryan at level II of the procedural layer. These same characteristics will make it difficult for him to grow or

Figure 6:18
Ryan's Worksheet H-4

COMPLETE THE PYRAMID

Study the following pyramid to see how the numbers are obtained.

Complete the pyramid by supplying the missing numbers.



0.01
0.02 0.02
0.03 0.04 0.03
0.04 0.07 0.07 0.04
0.05 0.11 0.14 0.11 0.05
0.6 0.16 0.25 025 0.16 0.6
0.7 0.22 0.41 0.50 0.41 0.22 0.7
0.8 0.29 0.63 0.91 0.91 0.63 0.29 0.08
0.9 0.37 0.92 1.54 1.82 1.54 0.92 0.37 0.9
0.10 0.45 1.29 2.46 3.36 3.36 2.46 1.29 0.45 0.10

peel the procedural layer. Yet there is evidence to show that the correct procedures could be reinforced.

Ryan scored 38% on the post-test which is below the post-test mean and almost exactly the pretest mean. It is difficult to say whether the teaching experiment helped Ryan or not as his post-test score and his delayed post-test scores cannot be compared because he quit school before the delayed post-test was administered.

PRESENTATION OF JENNIFER'S WORK

Jennifer completed very few of the worksheets (Figure 6:19) on her own. She would attempt a worksheet and either not finish it or would copy the answers from one of her groupmates. The following section summarized her performance.

Place Value

Jennifer began the teaching experiment by working with Paul. She did three of the place value worksheets (E-3,4,5) with Paul and then changed to another group. She attempted to complete these worksheets by herself and her responses (worksheets E-3,4,5) showed that she was able to apply certain procedures correctly but did not know why. For example, she answered the following two questions correctly but could not explain the reason for her answer.

1. CIRCLE the largest of these three numbers.
   (From worksheet E-4)

   [✓ 5.75] 5.485 5.5

   How did you decide that it was the largest?

   Response: It looks the biggest to me.

2. CIRCLE the larger of the two numbers.
   (From worksheet E-5)

   [✓ 10.9] or 10.75

   How do you know it is bigger?

   Response: I just know.

At other times Jennifer displayed a certain procedure that she thought was correct but did not use that reasoning all the way through the question. For example, in the

Figure 6:19
Jennifer's Worksheets

Place Value		Scale Reading		Fraction-to-Decimal Translation		Operations	
E-1	✓	F-1		G-1	A/C	H-1	A/C
E-2	✓	F-2	A/C	G-2	A/C	H-2	C
E-3	A/C	F-3		G-3	A/C	H-3	
E-4	A/C	F-4		G-4	A/C	H-4	A
E-5	A/C					H-5	
E-6	C					H-6	
E-7	A						
E-8	✓						
E-9	✓						
E-10	✓						
E-11	✓						

✓ Completed A Attempted C Copied

following question:

CIRCLE all the numbers that are BIGGER than 0.4
(From worksheet E-5)

[✓ 0.15] [✓ 0.09] [✓ 0.5] 0.104 0.3

Jennifer circled 0.15 and 0.09 because these numbers had two digits after the decimal point whereas 0.4 had only one digit after the decimal point. She reasoned that a number with two digits after the decimal point (no matter what the digit) was larger than a number with one digit after the decimal point. Yet, she did not circle 0.104 which had three digits after the decimal point. When asked why she had circled 0.5 she replied, "0.5 is half and that's the biggest of all the numbers in the list." She was unable to explain or elaborate any further.

She did not use this reasoning in question 4, worksheet E-4. Question 4 asked the following:

CIRCLE all the numbers that are BIGGER than 0.6

0.12 [✓ 0.8] 0.45 0.536 0.4.

In this case her response was correct. Had she used her previous procedure, she would have responded with 0.12, 0.45, and 0.536 which were all incorrect. It was brought to the teacher/researcher's attention that her correct responses had been copied from Paul.

The teacher/researcher observed that Jennifer was not interested in mathematics and copied the majority of her answers from other members of her group. Paul displayed confidence in his procedures and Jennifer was more willing to copy responses than to try to work them out herself. There was no discussion between Jennifer and Paul other than Jennifer asking Paul for certain answers. This might explain why Jennifer's answers did not follow a similar pattern.

Jennifer decided to change groups after the first day of the teaching experiment; she then joined Ryan, Tyler, and Julie. Jennifer was observed copying the answers to worksheet E-6 (Figure 6:20) from Ryan's paper. Ryan's responses were discussed in an earlier section.

When Jennifer was faced with a situation that she thought she knew, she would attempt a procedure. Most often the procedure was incorrect, yet it did somewhat resemble the correct procedure. For example, when it was necessary to place the representation of the expanded form of a decimal number on the abacus, she did not find the equivalent decimal number but instead placed each part of the expanded form on the abacus (Figure 6:21). The decimal point and all digits to the left of the

Figure 6:20
Jennifer's Worksheet E-6

1. Write the number that is one tenth greater.

a) 2.5 2.6
b) 1.9 1.10
c) 25.7 25.7
d) 0.68 0.69
e) 125.2 125.3
f) 0.7 0.8

2. Write the number that is one hundredth greater.

a) 8.75 8.76
b) 0.93 0.93
c) 1.085 1.086
d) 3.346 3.356
e) 17.591 17.590
f) 0.083 0.093

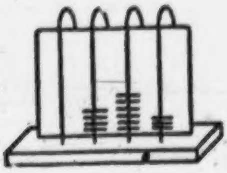
3. Write the number that is one thousandth less.

a) 5.238 5.239
b) 6.088 6.089
c) 0.13 0.14
d) 4.959 4.950
e) 6.781 6.782
f) 15.326 15.326

Figure 6:21
Jennifer's Worksheet E-7

DECIMALS AND THE ABACUS

46.3 can be shown on an abacus

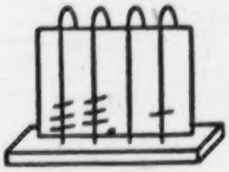


The number 46.3 is written below in expanded form.

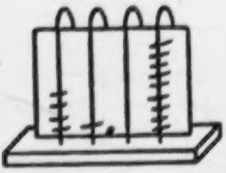
40 + 6 + 0.3

= 4 tens + 6 ones + 3 tenths
= (4x10) + (6x1) + (3x0.1)
= (4x10) + (6x1) + (3x1/10)

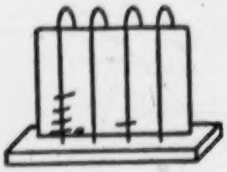
A.
Show each of the following expanded forms on the abacus.



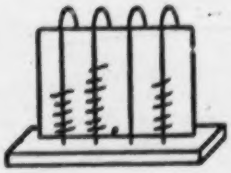
1. 30 + 3 + 0.1



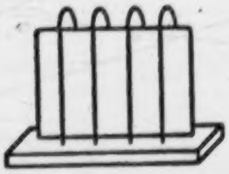
2. 40 + 1 + 0.9



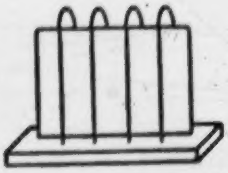
3. 4 + 0.1



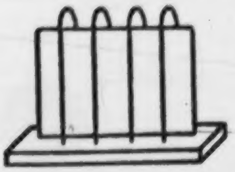
4. 50 + 7 + 0.5



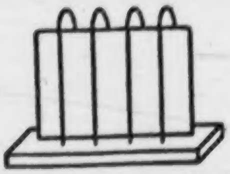
5. (5 x 4) + (3 x 1) + (2 x 0.1)



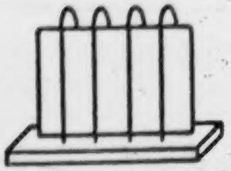
6. (4 x 10) + (4 x 1) + (2 x 0.1)



7. (3 x 10) + (5 x 1) + (4 x 0.1)



8. (3 x 10) + (5 x 1) + (2 x $\frac{1}{10}$)



9. (6 x 10) + (4 x 1) + (3 x $\frac{1}{10}$)

decimal point were correctly placed. However, the "zero" in the units place in front of the tenths digit was also given a space on the abacus. The items she completed were done in this mechanical manner.

Scale Reading

Jennifer completed only one worksheet on scale reading, worksheet F-2, of which she copied most of the answers from Julie's paper. Jennifer worked with only Julie on this worksheet as Ryan and Tyler were absent. Although Julie finished the other worksheets on scale reading, Jennifer did not, and she made no attempt to work with Julie. When asked why she did not work on the other worksheets, she replied that she "just didn't feel like doing anything today". While Julie was completing the worksheets, Jennifer was doodling on a piece of paper or talking to someone in another group.

Jennifer's responses to worksheet F-2 (Figure 6:22) matched Julie's and although Julie intermingled fractions with decimals, Jennifer did not see anything unusual with this procedure and accepted Julie's answers as correct. Jennifer did not ask Julie how she arrived at a response. Julie's responses were discussed in another section.

Fraction-to-Decimal Translation

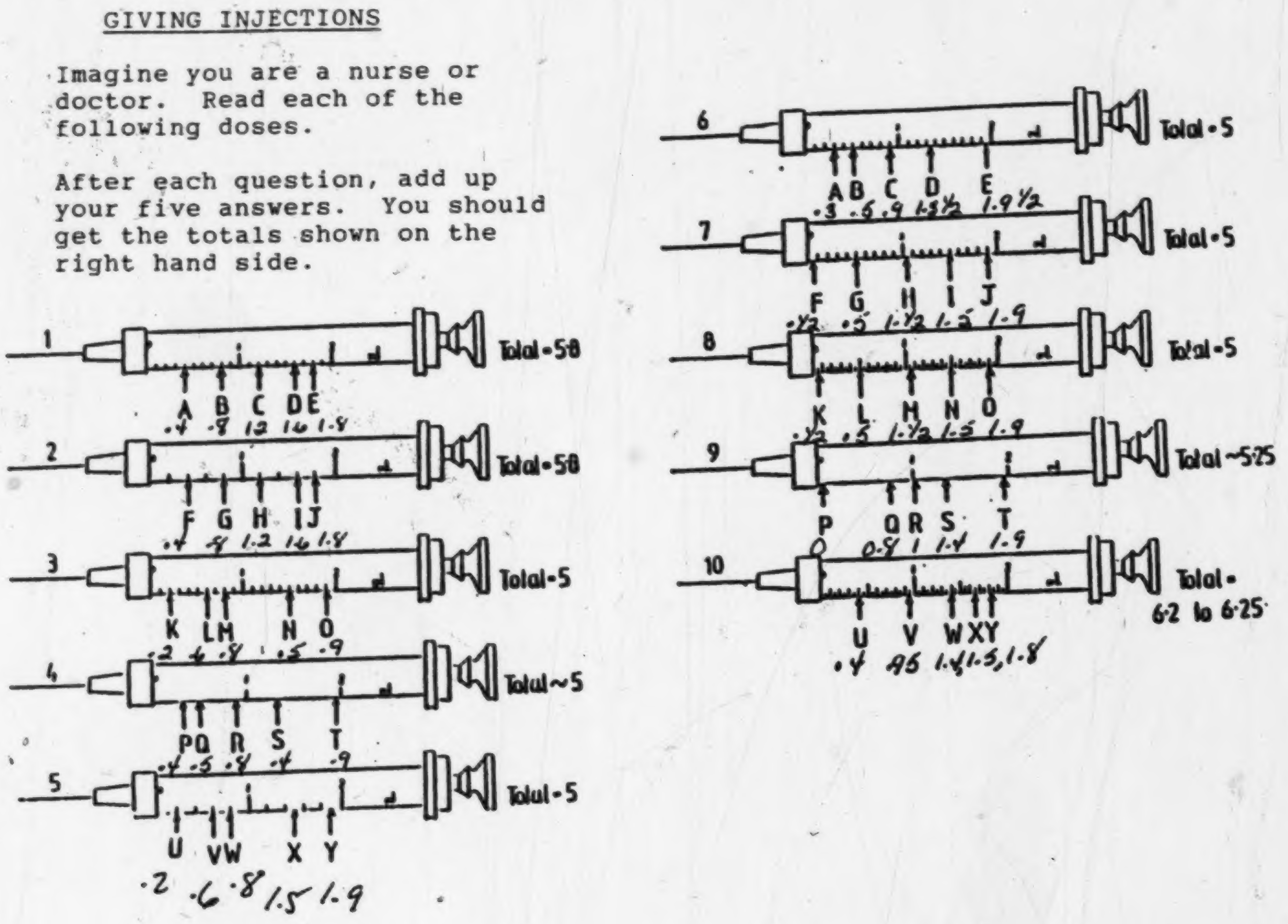
Jennifer completed four of the five worksheets on fraction-to-decimal translation. Jennifer's responses to worksheets G-1,2,3,4 were mostly correct and corresponded to the responses of the other members of the group. She relied on the other members of her group to provide her with the responses.

Operations

Jennifer did not complete all the operations' worksheets. She was absent one day and did not want to take them up the next day. She completed three of the six operations' worksheets H-1,2,4. Jennifer began worksheet H-1 (Figure 6:23) and completed items 1 to 5 on her own but copied items 6,7 and 8 from the group. Her responses to items 1 to 5 were incorrect once the response went from a decimal number with no whole unit, to one with whole units.

Jennifer was unable to find the pattern in worksheet H-4 (Figure 6:24). She observed that the numbers on the extreme left and extreme right sides of the pyramid should be exactly the same. The numbers on the right side of the pyramid were correct but the numbers on the left side were not. She observed that the numbers in those two columns were to go from one to ten but did not make a distinction between tenths and hundredths. Although she knew that the two columns should be exactly the same, she did not observe that her responses were not. The rest of the pattern was taken from another member of the group.

Figure 6:22
Jennifer's Worksheet F-2



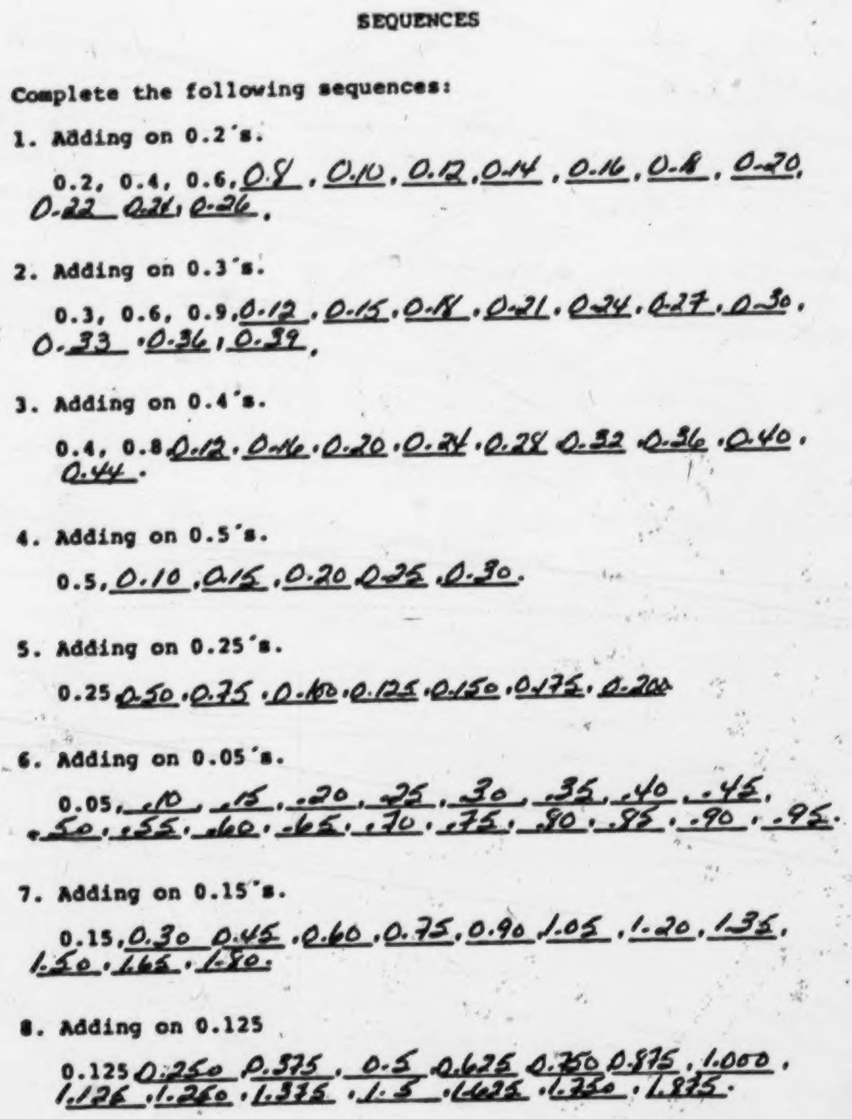
GIVING INJECTIONS

Imagine you are a nurse or doctor. Read each of the following doses.

After each question, add up your five answers. You should get the totals shown on the right hand side.

1
A B C DE
.4 .8 12 16 1.8
Total • 5.8
2
F G H IJ
.4 .8 1.2 16 1.8
Total • 5.8
3
K LM N O
.2 .4 .8 .5 .9
Total • 5
4
PQ R S T
.4 .5 .8 .4 .9
Total ~ 5
5
U VW X Y
.2 .6 .8 1.5 1.9
Total • 5
6
A B C D E
.3 .6 .9 1.3½ 1.9½
Total • 5
7
F G H I J
.½ .5 1.½ 1.5 1.9
Total • 5
8
K L M N O
.½ .5 1.½ 1.5 1.9
Total • 5
9
P Q R S T
0 0.8 1 1.4 1.9
Total ~ 5.25
10
U V W X Y
.4 .95 1.4, 1.5, 1.8
Total •
6.2 to 6.25

Figure 6:23
Jennifer's Worksheet H-1



SEQUENCES

Complete the following sequences:

1. Adding on 0.2's.

0.2, 0.4, 0.6, 0.8 , 0.10 , 0.12 , 0.14 , 0.16 , 0.18 , 0.20,
0.22 0.24 , 0.26 .

2. Adding on 0.3's.

0.3, 0.6, 0.9, 0.12 , 0.15 , 0.18 , 0.21 . 0.24 . 0.27 . 0.30 ,
0.33 . 0.36 , 0.39 ,

3. Adding on 0.4's.

0.4, 0.8 0.12 . 0.16 . 0.20 . 0.24 . 0.28 0.32 . 0.36 . 0.40 .
0.44 .

4. Adding on 0.5's.

0.5, 0.10 , 0.15 , 0.20 0.25 . 0.30 .

5. Adding on 0.25's.

0.25 0.50 . 0.75 . 0.100 . 0.125 . 0.150 . 0.175 . 0.200

6. Adding on 0.05's.

0.05, .10 , .15 , .20 , .25 , .30 , .35 , .40 , .45 ,
.50 , .55 , .60 , .65 , .70 , .75 , .80 , .85 , .90 , .95 .

7. Adding on 0.15's.

0.15, 0.30 0.45 , 0.60 , 0.75 , 0.90 1.05 , 1.20 , 1.35 ,
1.50 , 1.65 , 1.80 .

8. Adding on 0.125

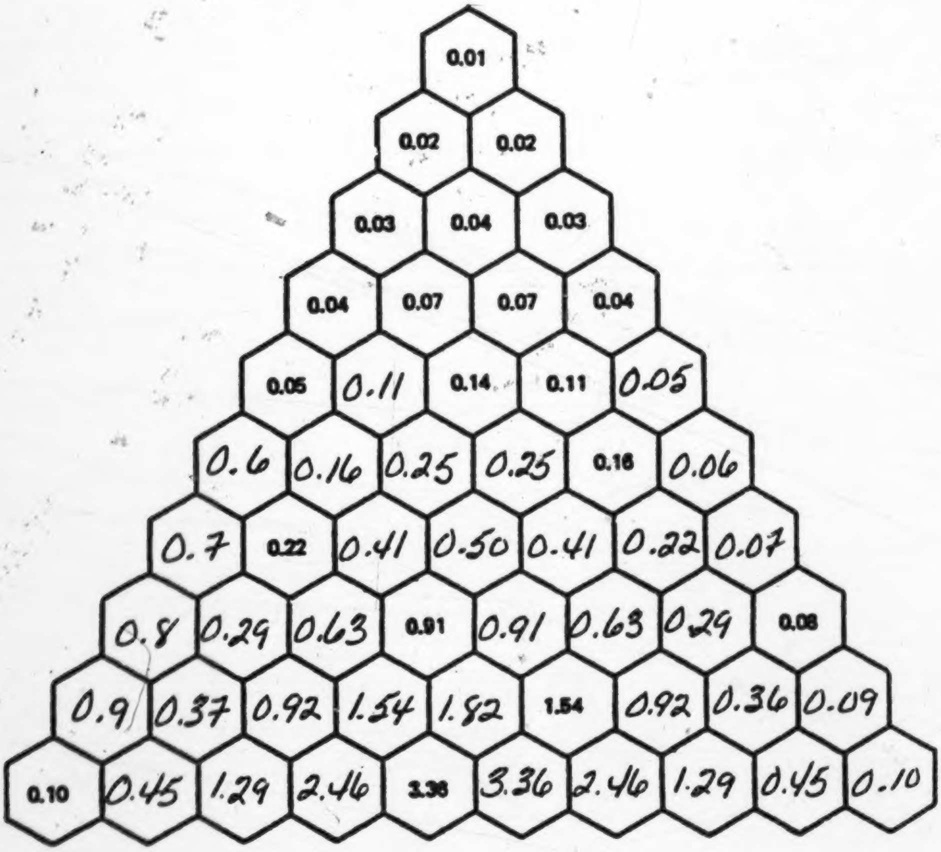
0.125 0.250 0.375 , 0.5 0.625 0.750 0.875 , 1.000 ,
1.125 . 1.250 . 1.375 . 1.5 . 1.625 . 1.750 . 1.875 .

Figure 6:24
Jennifer's Worksheet H-4

COMPLETE THE PYRAMID

Study the following pyramid to see how the numbers are obtained.

Complete the pyramid by supplying the missing numbers.



0.01
0.02 0.02
0.03 0.04 0.03
0.04 0.07 0.07 0.04
0.05 0.11 0.14 0.11 0.05
0.6 0.16 0.25 0.25 0.16 0.06
0.7 0.22 0.41 0.50 0.41 0.22 0.07
0.8 0.29 0.63 0.91 0.91 0.63 0.29 0.08
0.9 0.37 0.92 1.54 1.82 1.54 0.92 0.36 0.09
0.10 0.45 1.29 2.46 3.36 3.36 2.46 1.29 0.45 0.10

Interpretation of Jennifer's Work

It was difficult to assess Jennifer's performance as she did not actively participate in the discussion/solution process. She relied heavily on the other members of her group and copied most of the answers without any interest in the answer itself or whether it was correct or incorrect.

It was difficult to interest or motivate Jennifer. She worked when she felt like working and did not actively participate in the discussions with her groupmates.

Of the eight students taking part in the teaching experiment, Jennifer displayed the least correct procedural knowledge. Jennifer was one of the students who took part in the pretest interviews and the interview also revealed her lack of correct procedures.

Jennifer seemed to have developed few correct procedures and had no sense of what a decimal was or how to work with one. When relating decimal numbers to money she was still unable to perform the correct operations for the majority of the questions, but her responses improved slightly on addition and subtraction techniques

She appeared to be at a low level of the procedural layer. She was placed at level I of the procedural layer and it seemed that her lack of motivation and interest in mathematics and school would make it difficult for her to peel the procedural layer. It might be possible to reteach her the correct procedures so that she could function efficiently at the procedural layer within the system of decimal numbers but it was felt that her decimal number knowledge would be limited.

Jennifer scored 32% on the post-test which was below the post-test mean and below the pretest mean. This was also an indication of incorrect procedural knowledge. Yet, on the delayed post-test she scored 58% which was above the delayed post-test mean. It seems incredible that Jennifer's scores increased to this extent between the post-test and the delayed post-test.

PRESENTATION OF JULIE'S WORK

Julie completed nearly all the worksheets (Figure 6:25). She was also one of the few students who was never absent and was always on time. The following section summarized her performance.

Place Value

Julie was able to take effective action when dealing with the place value of decimal numbers. She was able to use a specific place value position when comparing decimal numbers (Figure 6:26). She specified that the tenths position was the position to be examined (item 1). She was one of the few students who used specific

Figure 6:25
Julie's Worksheets

Place Value		Scale Reading		Fraction-to-Decimal Translation		Operations	
E-1	✓	F-1	✓	G-1	✓	H-1	✓
E-2	✓	F-2	✓	G-2	✓	H-2	✓
E-3	✓	F-3		G-3	✓	H-3	✓
E-4	✓	F-4	✓	G-4	✓	H-4	✓
E-5	✓					H-5	✓
E-6	✓					H-6	✓
E-7	✓						
E-8	✓						
E-9	✓						
E-10	✓						
E-11	✓						

✓ Completed A Attempted C Copied

Figure 6:26
Julie's Worksheet E-4

COMPARING DECIMALS

1. CIRCLE the largest of these three numbers:

[✓ 5.75]   5.485   5.5

How did you decide that it was the largest?
Because the number in the tenths position is the largest.

2. Write the following numbers in ascending order (SMALLEST first).

3.87	3.9	4.07	4.5	3.49	3.785	3.6
4	5	6	7	1	3	2

3. In each list CIRCLE the two numbers that are equal:

a) [✓ 78]   780   [✓ 078]   708

b) [✓ 0.150]   [✓ 0.15]   0.015   0.105

4. CIRCLE all the numbers that are BIGGER than 0.6:

0.12   [✓ 0.8]   0.45   0.536   0.4

CIRCLE all the numbers that are SMALLER than 0.75:

[✓ 0.6]   [✓ 0.125]   [✓ 0.096]   0.8   [✓ 0.106]

5. Write down, if possible, any number that is BIGGER than the first, but SMALLER than the second:

a) 3.5 < 3.6 <3.7
b) 3.9 < 3.10 <3.11
c) 4.1 < 4.15 <4.20
d) 5.00 < 5.05 <5.10
e) 5.100 < 5.105 <5.11

vocabulary. She correctly ordered decimal numbers (items 2 and 3) and correctly placed a decimal number between two given decimal numbers (item 5). This sometimes involved not only the tenths position but the hundredths and the thousandths position as well. Her procedure (items 5c,5d,5e) was to add zeros to the number until each decimal number had the same number of digits after the decimal point. She was then able to place correctly a decimal number between the two given numbers. Her procedural knowledge allowed her to place as many zeros after the last digit in a decimal number and not change the value of the decimal number. She used this rule to help her with the ordering of decimals. She would first compare the digits in the tenths place; if they were equal she would then compare the digits in the hundredths place, etc., until all the decimal numbers had been placed in order.

Julie correctly identified tenths, hundredths and thousandths to complete worksheet E-6 (Figure 6:27). Ryan asked her how to do this particular worksheet. She replied, "Just add one to the last number." When Ryan asked the question, Julie had completed the first three items of the question. Up to this point that particular procedure gave a correct response. However, this rule did not apply to all the items. Julie's correct worksheet E-6 (except for item 2e) demonstrated that she was able to distinguish place value positions and she changed her rule to answer correctly the other items on the worksheet.

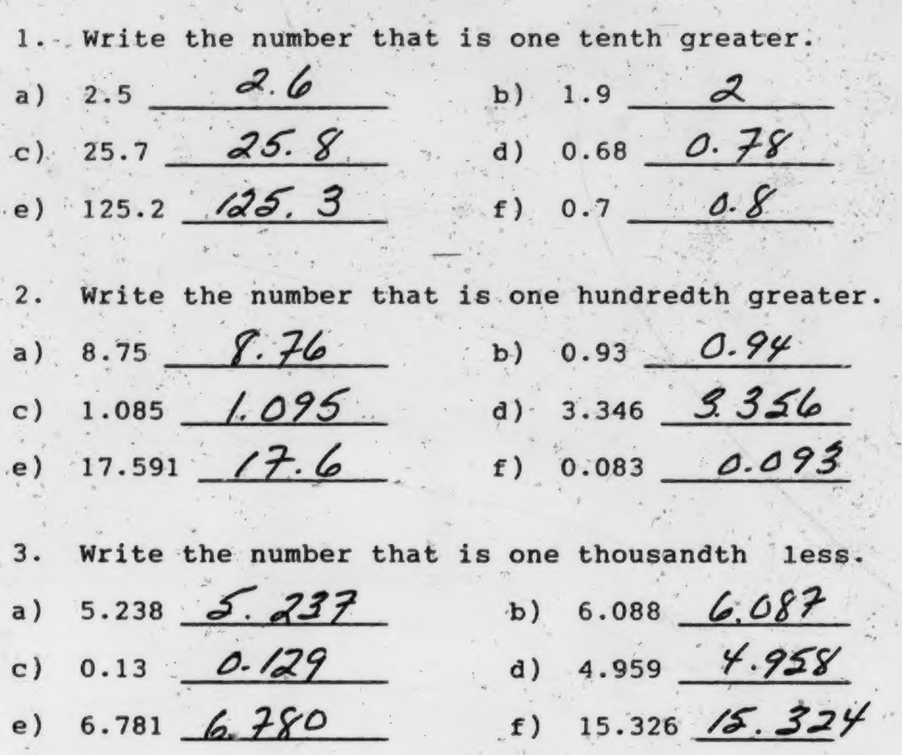
In order to represent a decimal number in expanded form on the abacus, Julie simplified the expanded form and then placed its representation on the abacus (Figure 6:28). By simplifying the expanded form, she was better able to distinguish the tens, units and tenths. This resulted in a correct worksheet.

Julie's group worked together to complete worksheet E-10 (Figure 6:29). "The Fewest Number of Coins" required that the students find the smallest number of coins or bills to make each given amount. They found this easy and took turns completing a row.

Scale Reading

Julie had difficulty with the scale reading worksheets. She was able to read scales subdivided into fifths and tenths but could not estimate a reading between marked calibrations. When she was asked to read between marked calibrations, her answer contained a fraction. Figure 6:30 (worksheet F-2) showed this process. The same procedure was applied to worksheet F-4 (Figure 6:31). When Julie was required to estimate a reading between marked calibrations, her response usually consisted of a fraction combined with a decimal number. For example, item d (Figure 6:31) was halfway between 3 and 0.1 and therefore Julie responded with 3.1/2. She correctly read items a,b,c which were on a number line divided into tenths, and correctly read the items which were on a number line divided into fifths (items, e,f,g,h). Items i,j,k,l were on a number line divided into twentieths and Julie could read correctly the items

Figure 6:27
Julie's Worksheet E-6



1. Write the number that is one tenth greater.

a) 2.5 2.6
b) 1.9 2
c) 25.7 25.8
d) 0.68 0.78
e) 125.2 125.3
f) 0.7 0.8

2. Write the number that is one hundredth greater.

a) 8.75 8.76
b) 0.93 0.94
c) 1.085 1.095
d) 3.346 3.356
e) 17.591 17.6
f) 0.083 0.093

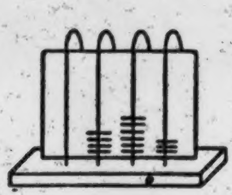
3. Write the number that is one thousandth less.

a) 5.238 5.237
b) 6.088 6.087
c) 0.13 0.129
d) 4.959 4.958
e) 6.781 6.780
f) 15.326 15.324

Figure 6:28
Julie's Worksheet E-7

DECIMALS AND THE ABACUS

46.3 can be shown on an abacus

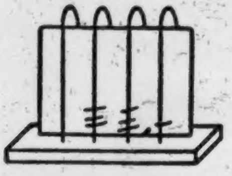


The number 46.3 is written below in expanded form.

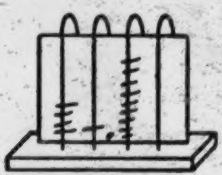
40 + 6 + 0.3

= 4 tens + 6 ones + 3 tenths
= (4x10) + (6x1) + (3x0.1)
= (4x10) + (6x1) + (3x1/10)

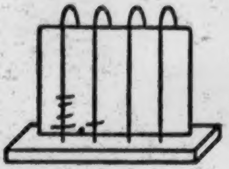
A.
Show each of the following expanded forms on the abacus.



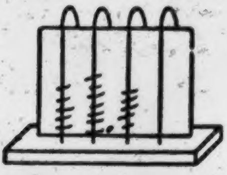
1. 30 + 3 + 0.1 33.1



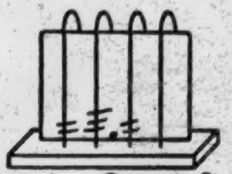
2. 40 + 1 + 0.9 41.9



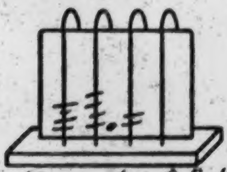
3. 4 + 0.1 4.1



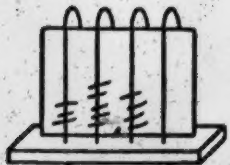
4. 50 + 7 + 0.5 57.5



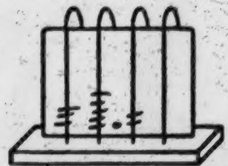
20 3 0.2 = 23.2
5. (5 x 4) + (3 x 1) + (2 x 0.1)



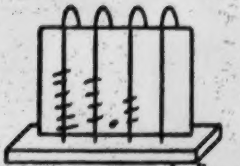
40 4 0.2 = 44.2
6. (4 x 10) + (4 x 1) + (2 x 0.1)



30 5 0.4 = 35.4
7. (3 x 10) + (5 x 1) + (4 x 0.1)



30 5 .2 = 35.2
8. (3 x 10) + (5 x 1) + (2 x $\frac{1}{10}$)



60 4 0.3 = 64.3
9. (6 x 10) + (4 x 1) + (3 x $\frac{1}{10}$)

Figure 6:29
Worksheet E-10

THE FEWEST NUMBER OF COINS

What is the fewest number of coins or bills needed to make each of the following amounts? Record your answers in the spaces provided. The first one is done for you.

A.

	$1.00	$0.50	$0.25	$0.10	$0.05	$0.01	Total Number
$0.07					/	//	3
$0.09					/	////	5
$0.13				/		///	4
$0.28			/			///	4
$0.59		/			/	////	6
$0.75		/	/				2
$2.78	//	/	/			///	7
$3.61	///	/		/		/	6
$5.51	/////	/				/	7
$6.03	//////					///	9

B.

	$5.00	$2.00	$1.00	$0.50	$0.25	$0.10	$0.1	Total Number
$2.98		/		/	/	//	///	8
$3.96		/	/	/	/	//	/	7
$5.97	/			/	/	//	//	7
$6.97	/		/	/	/	//	//	8
$9.98	/	//		/	/	//	///	10
$7.98	/	/		/	/	//	///	9
$8.92	/	/	/	/	/	/	///////	13
$10.01	//						/	3
$12.24	//	/				//	////	9
$10.99	//			/	/	//	////	10

GIVING INJECTIONS

Imagine you are a nurse or doctor. Read each of the following doses.

After each question, add up your five answers. You should get the totals shown on the right hand side.

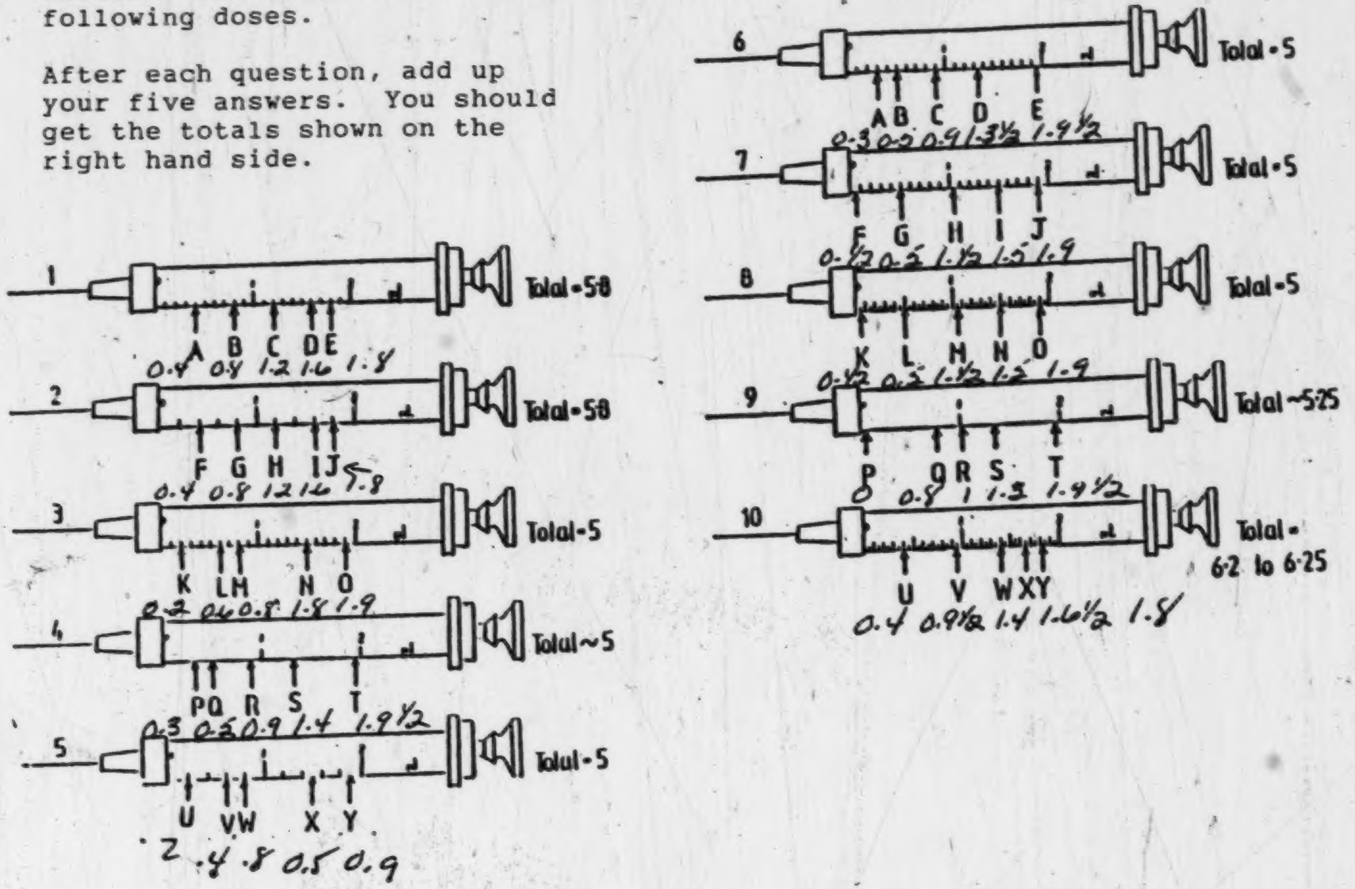
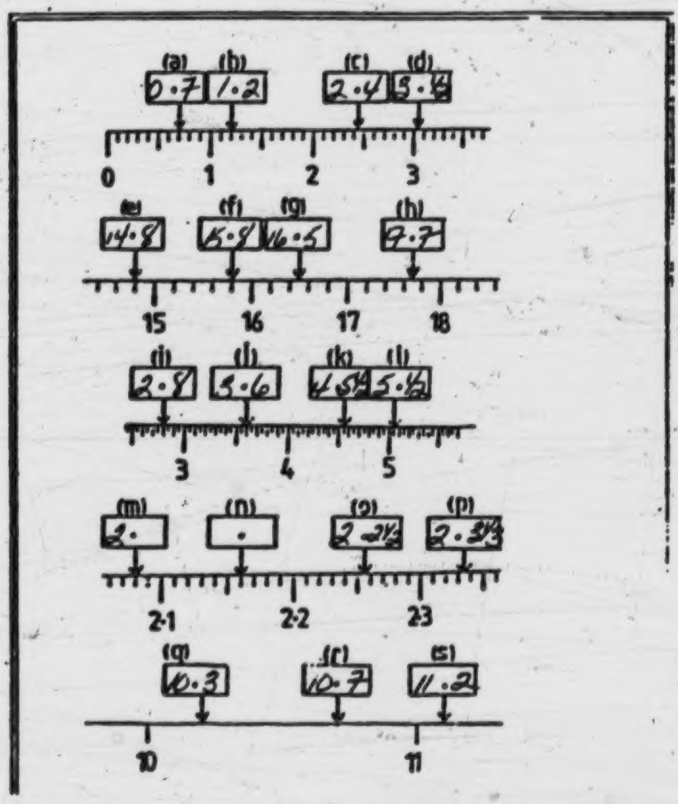

1
A B C DE
0.4 0.8 1.2 1.6 1.8
Total • 5·8
2
F G H IJ
0.4 0.8 1.2 1.6 1.8
Total • 5·8
3
K LM N O
0.2 0.4 0.8 1.8 1.9
Total - 5
4
PQ R S T
0.3 0.5 0.9 1.4 1.9½
Total ~ 5
5
U VW X Y
Z .4 .8 0.5 0.9
Total - 5
6
AB C D E
0.3 0.5 0.9 1.3½ 1.9½
Total • 5
7
F G H I J
0.½ 0.5 1.½ 1.5 1.9
Total • 5
8
K L M N O
0.½ 0.5 1.½ 1.5 1.9
Total - 5
9
P Q R S T
0 0.8 1 1.3 1.9½
Total ~5·25
10
U V WXY
0.4 0.9½ 1.4 1.6½ 1.8
Total • 6·2 to 6·25

Figure 6:30
Julie's Worksheet F-2

Figure 6:31
Julie's Worksheet F-4

FURTHER PRACTICE SHEET

Read the following scales and write your answers in the spaces provided.


0
1
2
3
15
16
17
18
3
4
5
2·1
2·2
2·3
10
11

(a) 0·7
(b) 1·2
(c) 2·4
(d) 3·½
(e) 14·8
(f) 15·8
(g) 16·5
(h) 7·7
(i) 2·8
(j) 3·6
(k) 4·5½
(l) 5·½
(m) 2·
(n) ·
(o) 2·2½
(p) 2·3½
(q) 10·3
(r) 10·7
(s) 11·2

which were on a 'tenth' mark (items i,j) but could not read correctly the items which were not on a 'tenth' mark (k,l). With these items, she used the fraction 1/2 to indicate that the arrow marked a point that was halfway between tenth marks. This same procedure was used for worksheet F-2 (Figure 6:30). The number line divided into hundredths (items, m,n,o,p) also proved difficult for Julie.

Julie did not attempt worksheet F-3, Marking Homework. On this particular day Ryan and Tyler were absent which left Julie and Jennifer in the group. They were asked by the teacher/researcher if they wanted to join another group for that day and they both said no. After having completed two of the worksheets, with no help from Jennifer, Julie became frustrated and said, "I've done enough for one day. I'm not doing any more." After this statement was made she refused even to look at the worksheet. She kept asking if she could leave the room to go outside and smoke or go to the washroom. She spent the rest of the class time talking to her friends. This happened to be the beginning of a long weekend and motivation was low. The approaching long weekend was much more interesting than mathematics.

Fraction-to-Decimal Translation

When Julie was given a pack of cards (Figure 6:6) and was told that she was to make equivalent triplets she said, "Oh, this could be fun." It became a game and she became engrossed in the task and correctly identified all triplets. She was questioned a few times by Ryan who was unsure of his answers.

Julie's responses to the worksheets dealing with fraction-to-decimal translation were well done. Julie was able to interchange fractions, decimals and percents. The task of changing a percent to a decimal and vice versa was a mechanical process. Her procedural knowledge told her that she could go from one to the other by moving the decimal point two places to the left or right.

Julie's procedural knowledge needed reinforcing at times. She was aware that procedures existed to help her work with decimal numbers but in certain instances, she was not sure whether her procedures were correct. She thus needed reassurance. For example, when translating fractions to decimals she divided the denominator into the numerator but was not sure this was correct. She asked the teacher/researcher, "I don't remember how to do this. Is this right?" She was recalling a learned procedure and needed to know if her recalled procedure was correct.

Operations

When faced with a written problem where a choice had to be made as to which operation to perform, Julie became frustrated and impatient when she did not know which operation to select. She had no difficulty choosing the operations of addition or subtraction, but the operations of multiplication and division were more difficult. Once the operation had been chosen she had no difficulty carrying it out. For example, Julie had trouble deciding whether an answer should be more or less than the number given in the problem. Item 5, worksheet H-6, required the student to calculate 84% of 68 300. The question reads:

> A football stadium has seating for 68 300 people. If 84% of the seats are occupied, how many people are at the football game?

She was unable to tell the teacher/researcher that the answer should be less that 68 300. The teacher/researcher asked Julie, "How many seats would be filled if 100% of the seats were occupied?"

J: All of them.
T/R: Is 84% more or less than 100%?
J: Less.
T/R: Is having 84% of the seats filled more or less than having 100% of the seats filled?
J: Less, so the answer would be less than 68 300.

Once Julie had decided that the answer would be less than the given number, she decided the operation should be division because "division makes smaller". The operations of multiplication and division of decimal numbers seemed to be confounded by the fact that multiplication and division of decimal numbers did not follow the rule "multiplication makes bigger and division makes smaller". Bell (1986) found that one of the chief misconceptions of students was the assumption that multiplication made bigger and division made smaller.

The percentage problems that dealt with money seemed to be easier to interpret than the percentage problems that did not deal with money. Julie could not interpret the previous problem dealing with seating at a football stadium but she was able to interpret correctly problems dealing with tax on a bought item. For example, the following question was asked on worksheet G-3:

> A stereo costs $578.50. The sales tax is 7%. How much sales tax would
> be added to the price of the stereo?
> What is the cost of the stereo?

In this case she knew that the answer had to be more than $578.50 so she multiplied by 7% and added her answer to $578.50. In this case, her thinking of "multiplication makes bigger" gave her a correct response although it was the addition that made bigger and not the multiplication.

Julie did not know what the word perimeter meant (item 1, worksheet H-3). When discussing this question with the other members of her group she said, "I don't know what that word means. I think it means l x w". Julie had remembered the formula for area of a rectangle but did not know what it was she had remembered. Jennifer and Tyler did not offer any suggestions but Ryan shouted, "I know what it means. It means all around. We add them all up". The answer was easy to find after this revelation and she accepted Ryan's response as correct. This was a change for this group. Usually the group looked to Julie for a response. In this case, Ryan was the individual who displayed more confidence in his response and therefore his response was accepted as the correct one.

Julie was not able to "Complete the Pyramid", worksheet H-4, (Figure 6:32) without help from Tyler and the teacher/researcher. She looked at the diagram quickly and said, "I can't do this. How do you do this?"

T/R: Do you see any similarities in the diagram?
J: Yes. The numbers on the sides are the same.

Having said this, she quickly filled in the numbers on the left and right sides of the pyramid. However, she did not pay attention to the place value and used tenths rather than hundredths. Having completed these two columns, she then asked Tyler how to fill in the rest, not being able to see the pattern right away. He explained the procedure to her and she then completed the pyramid.

Interpretation of Julie's Work

Julie displayed good procedural knowledge that allowed her to take effective action when faced with a situation involving decimal numbers. She was able to perform the operations without difficulty.

Julie's group members looked to her for advice. They turned to her for the response or the procedure and did not question her responses. Julie was regarded as being the person in the group who was better at doing maths. However, when different responses were evident Julie would question other members. She would say, "How did you get that?", and the responses would be discussed. The group seemed to go along with those individuals who seemed more confident of their answers and in this group that one person was Julie.

Julie had difficulty interpreting the written problems that required multiplication or division. She would ask the teacher/researcher, "How do you do this?" and became very frustrated and impatient when the teacher/researcher would ask questions such as, "Should the answer be smaller or larger?" rather than giving her the procedure directly. In this case Julie said, "I want you to tell me how. I don't want to know why." Julie was interested in procedures and, depending on her mood that particular day, was not open to questioning from the teacher/researcher.

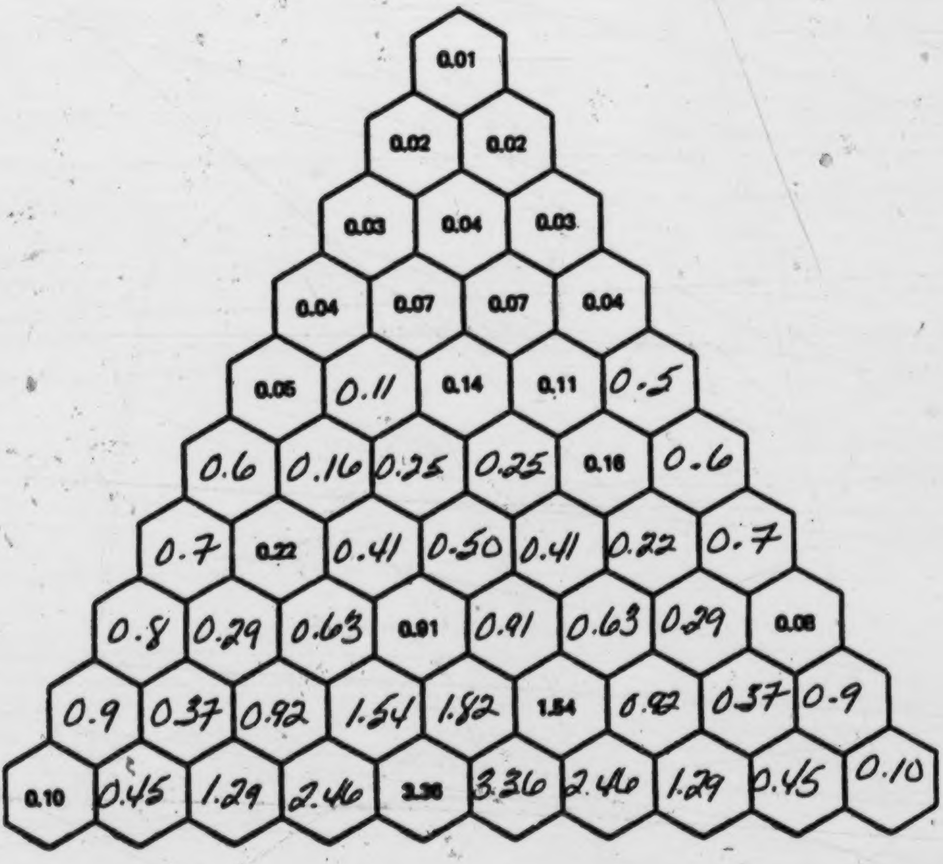
Julie could be placed at level III of the procedural layer. It was felt that Julie's

Figure 6:32
Julie's Worksheet H-4

COMPLETE THE PYRAMID

Study the following pyramid to see how the numbers are obtained.

Complete the pyramid by supplying the missing numbers.



0.01
0.02 0.02
0.03 0.04 0.03
0.04 0.07 0.07 0.04
0.05 0.11 0.14 0.11 0.5
0.6 0.16 0.25 0.25 0.16 0.6
0.7 0.22 0.41 0.50 0.41 0.22 0.7
0.8 0.29 0.63 0.91 0.91 0.63 0.29 0.08
0.9 0.37 0.92 1.54 1.82 1.54 0.92 0.37 0.9
0.10 0.45 1.29 2.46 3.36 3.36 2.46 1.29 0.45 0.10

good procedural knowledge could be built upon so that she could peel the procedural layer. However, her insistence on knowing how a procedure works may be strong enough to inhibit her learning of the why a procedure works. If this is the case she would remain at the procedural layer.

Julie's knowledge of correct procedures was demonstrated on the post-test (78%) and on the delayed post-test (76%). Both scores were significantly above the post-test and delayed post-test means.

PRESENTATION OF PAUL'S WORK

Paul was the only student taking part in the teaching experiment to complete all the worksheets (Figure 6:33). He was also one of the students who was never absent and who was always on time. His performance was discussed in the following sections.

Place Value

Although Paul made references to specific place value positions during his interactions with the teacher/researcher, he did not do so on item 1 from worksheet E-4 (Figure 6:34). Other students made reference to the number in the tenths position or the number after the decimal, but Paul simply stated that ".7 is larger than .4 or .5." He seemed to be making reference to the digit in the tenths position but did not state this specifically. This seemed strange for Paul since he demonstrated in the pretest interviews that he could label the place value positions and could use these positions to order correctly decimal numbers.

Paul correctly identified decimal numbers that would be placed between two given numbers so that the three numbers would be in order. When discussing his responses with the teacher/researcher, he made reference to the place value positions of the digits. He made the comment that when comparing or ordering decimals, "if the tenths are the same, then you look at the hundredths, and if the hundredths are the same then you look at the thousandths and so on". He also said that a person could "add zeros and then compare the numbers because zeros at the end of a decimal number don't really count". This knowledge served him well in all cases except item 5e (Figure 6:34). This was his only error on the place value worksheets.

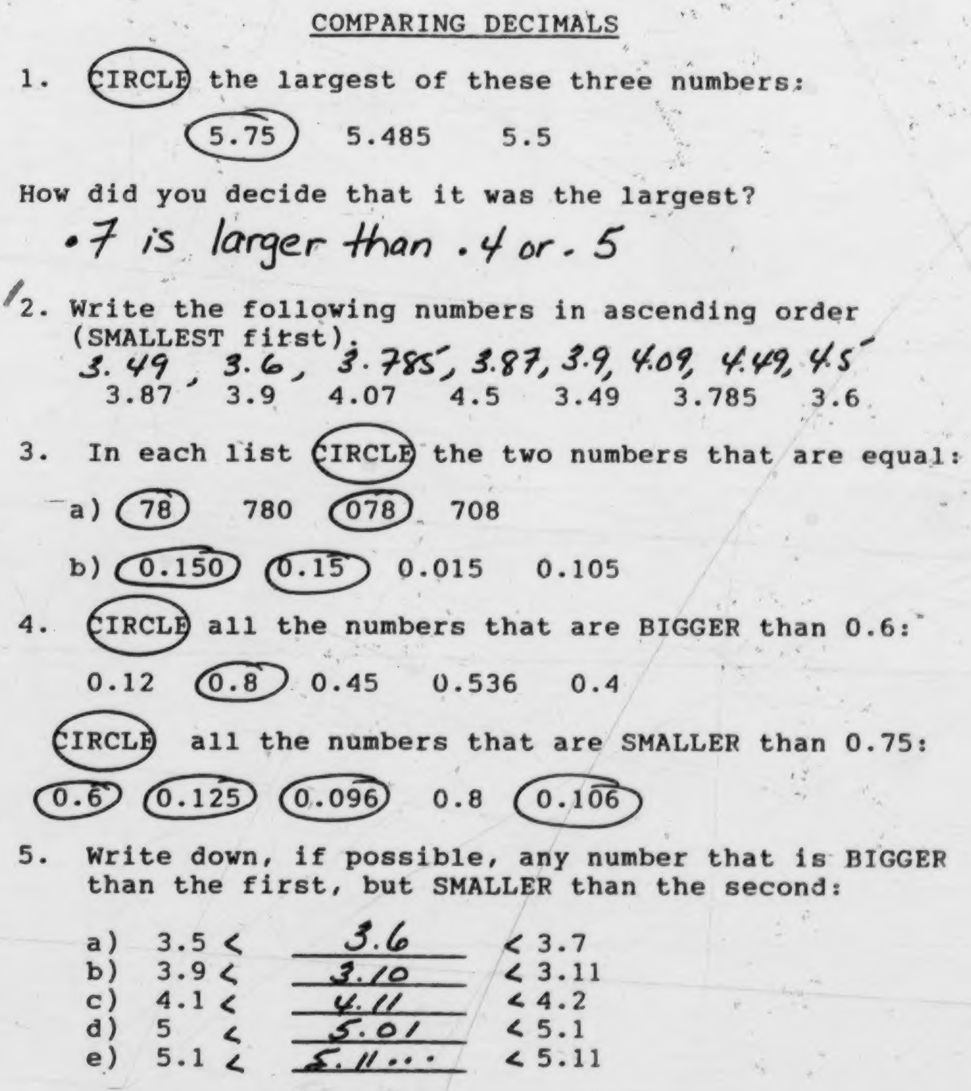
Paul was the one student taking part in the study who did not complete worksheet E-7 (Figure 6:35) by simplifying the expanded form of the decimal number. Paul took the expanded form given and rewrote it in a different expanded form. For example, rather than simplify the expanded form of 30 + 3 + 0.1 into 33.1, Paul took each separate term and wrote it in a different expanded form, (3 x 10) + (3 x 1) + (1 x 1/10). Paul noticed the different expanded forms of a decimal number at the top of the worksheet and chose the one which best suited his needs. He was the only student who noticed or paid any attention to the entire worksheet. The other students just wanted to

Figure 6:33
Paul's Worksheets

Place Value		Scale Reading		Fraction-to-Decimal Translation		Operations	
E-1	✓	F-1	✓	G-1	✓	H-1	✓
E-2	✓	F-2	✓	G-2	✓	H-2	✓
E-3	✓	F-3	✓	G-3	✓	H-3	✓
E-4	✓	F-4	✓	G-4	✓	H-4	✓
E-5	✓					H-5	✓
E-6	✓					H-6	✓
E-7	✓						
E-8	✓						
E-9	✓						
E-10	✓						
E-11	✓						

/ Completed A Attempted C Copied

Figure 6:34
Paul's Worksheet E-4



COMPARING DECIMALS

1. CIRCLE the largest of these three numbers:

[✓ 5.75] 5.485 5.5

How did you decide that it was the largest?

.7 is larger than .4 or .5

2. Write the following numbers in ascending order (SMALLEST first).

3.49, 3.6, 3.785, 3.87, 3.9, 4.09, 4.49, 4.5

3.87 3.9 4.07 4.5 3.49 3.785 3.6

3. In each list CIRCLE the two numbers that are equal:

a) [✓ 78] 780 [✓ 078] 708

b) [✓ 0.150] [✓ 0.15] 0.015 0.105

4. CIRCLE all the numbers that are BIGGER than 0.6:

0.12 [✓ 0.8] 0.45 0.536 0.4

CIRCLE all the numbers that are SMALLER than 0.75:

[✓ 0.6] [✓ 0.125] [✓ 0.096] 0.8 [✓ 0.106]

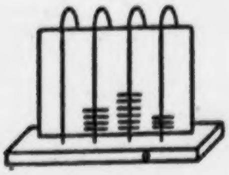
5. Write down, if possible, any number that is BIGGER than the first, but SMALLER than the second:

a) 3.5 < 3.6 < 3.7
b) 3.9 < 3.10 < 3.11
c) 4.1 < 4.11 < 4.2
d) 5 < 5.01 < 5.1
e) 5.1 < 5.11... < 5.11

Figure 6:35
Paul's Worksheet E-7

DECIMALS AND THE ABACUS

46.3 can be shown on an abacus

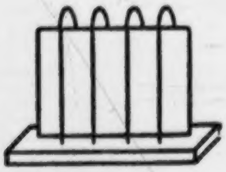


The number 46.3 is written below in expanded form.

40 + 6 + 0.3

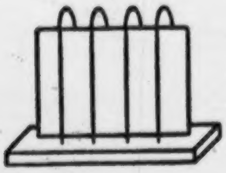
= 4 tens + 6 ones + 3 tenths
= (4x10) + (6x1) + (3x0.1)
= (4x10) + (6x1) + (3x1/10)

A.
Show each of the following expanded forms on the abacus.



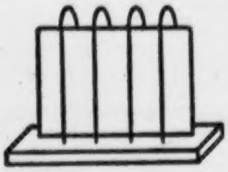
1. 30 + 3 + 0.1

$(3\times10)+(3\times1)+(1\times\frac{1}{10})$



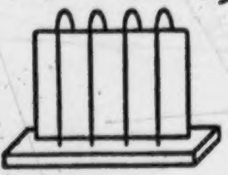
2. 40 + 1 + 0.9

$(4\times10)+(1\times1)+(9\times\frac{1}{10})$



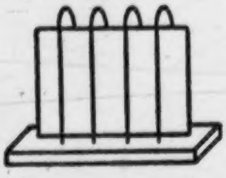
3. 4 + 0.1

$(4\times1)+(1\times\frac{1}{10})$



4. 50 + 7 + 0.5

$(5\times10)+(7\times1)+(5\times\frac{1}{10})$

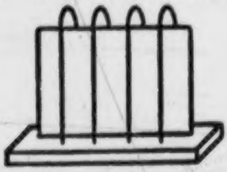


5. (5 x 4) + (3 x 1) + (2 x 0.1)

$(2\times10)+(3\times1)+(2\times\frac{1}{10})$

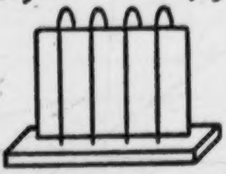
6. (4 x 10) + (4 x 1) + (2 x 0.1)

$(4\times10)+(4\times1)+(2\times\frac{1}{10})$



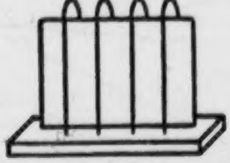
7. (3 x 10) + (5 x 1) + (4 x 0.1)

$(3\times10)+(5\times1)+(4\times\frac{1}{10})$



8. (3 x 10) + (5 x 1) + (2 x $\frac{1}{10}$)

$(3\times10)+(5\times1)+(2\times\frac{1}{10})$



9. (6 x 10) + (4 x 1) + (3 x $\frac{1}{10}$)

$(6\times10)+(4\times1)+(3\times\frac{1}{10})$

know how to complete an item so that they could do it and be done with it.

Paul's work with place value was very well done. He was able to specify place value positions in his responses. When he compared decimal numbers he stated that, "tenths are bigger than hundredths because one hundred is ten times ten so something cut into hundredths is a lot smaller than something cut into tenths". This implied that Paul had a notion of the place value system.

Scale Reading

Paul was able to read scales subdivided into units other than tenths and could estimate a reading which lay between marked calibrations.

Worksheet F-4 (Figure 6:36) was well done with only one error, item (a). Paul responded with 7.0 rather than 0.7. Upon being told that this answer was incorrect, he looked at the number line and quickly said, "Oh yeah! It should be .7". He correctly responded to the other items. Paul was the only student taking part in the teaching experiment to use correctly zero as a place holder (items d,l, and m) and interpret the number line to the thousandths place (items o and p).

Paul did excellent work on worksheet F-4 and F-2 (Figure 6:37). On worksheet F-3 (Figure 6:38), Paul identified eleven of the twelve errors he missed one and correctly changed eight of them. He changed item (c) from 2.1/2 to 2.5 but the correct response was 2.05. He knew that 2.1/2 was incorrect and automatically changed the fraction 1/2 to the decimal .5, which in this case led to an incorrect answer, without examining the number line. The same procedure was used in item (d). The fraction 1/2 was changed to a 5 which in this case gave a correct response. Substituting 5 for the fraction 1/2 also gave a correct response for item (n) but not for (g). The procedure of changing the fraction 1/2 to a 5 was not used in item (h). Here Paul correctly changed 6.1/2 to 6.1. Paul's response to (f) was correct but his response to (e) was incorrect although both items used the same scale.

Paul demonstrated previously that he could correctly read scales subdivided into fifths (items e,f,g,h; Figure 6:36 and question 2; Figure 6:37). The difference here may be that he had to do more than read the scale. Worksheet F-3 (Figure 6:38) required that the student examine Eric's responses, judge them as correct or incorrect and then correct the incorrect responses. It was felt by the teacher/researcher that Paul's performance would have been excellent had he been able to read the scale directly and fill in the response himself.

Fraction-to-Decimal Translation

Paul's worksheets dealing with fraction-to-decimal translation were well done. Paul was able to interchange fractions, decimals and percents. He knew a procedure to

Figure 6:36
Paul's Worksheet F-4

FURTHER PRACTICE SHEET

Read the following scales and write your answers in the spaces provided.

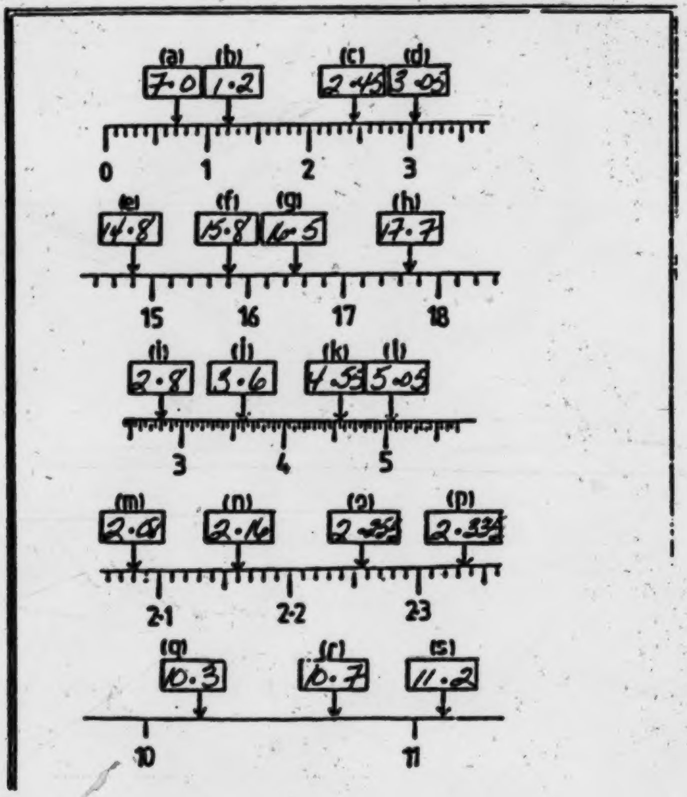
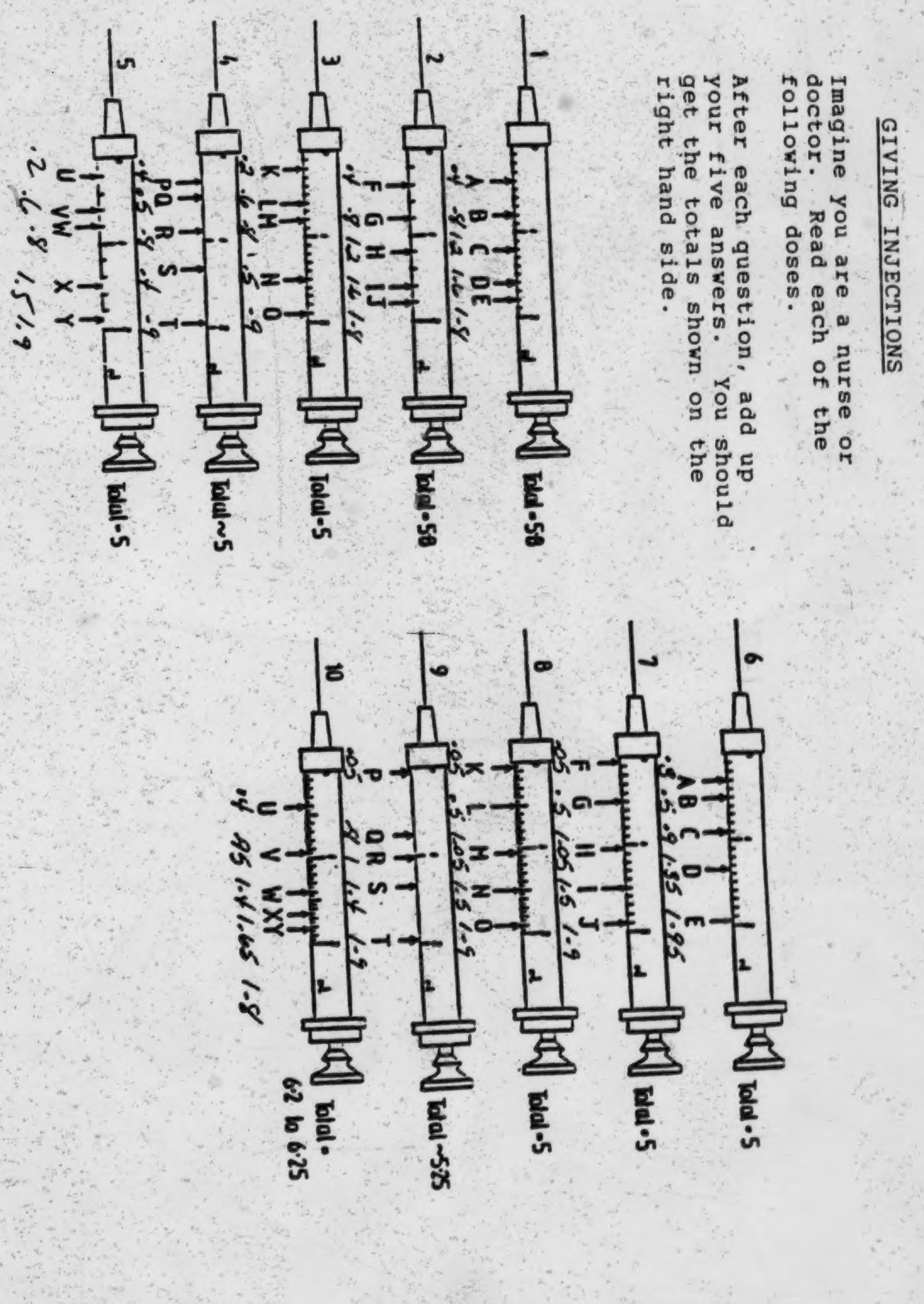

(a) 7·0
(b) 1·2
(c) 2·45
(d) 3·05
0
1
2
3
(e) 14·8
(f) 15·8
(g) 16·5
(h) 17·7
15
16
17
18
(i) 2·8
(j) 3·6
(k) 4·55
(l) 5·05
3
4
5
(m) 2·08
(n) 2·16
(o) 2·25
(p) 2·33½
2·1
2·2
2·3
(q) 10·3
(r) 10·7
(s) 11·2
10
11

Figure 6:37
Paul's Worksheet F-2



GIVING INJECTIONS

Imagine you are a nurse or doctor. Read each of the following doses.

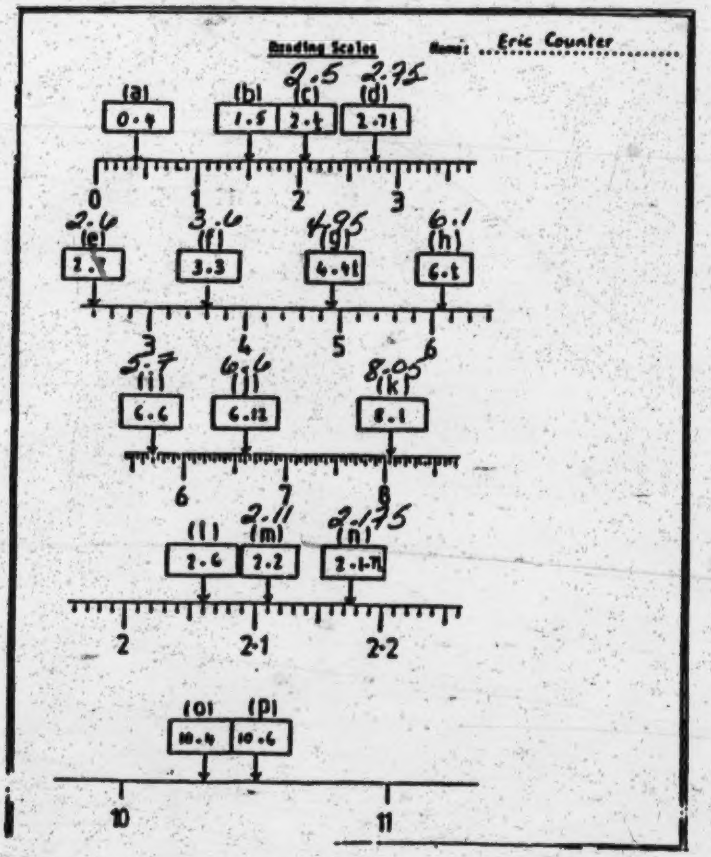
After each question, add up your five answers. You should get the totals shown on the right hand side.

1
A B C DE
.4 .8 1.2 1.6 1.8
Total - 5.8
2
F G H IJ
.4 .8 1.2 1.6 1.8
Total - 5.8
3
K LM N O
.2 .6 .8 1 .5 .9
Total - 5
4
PQ R S T
.1 .15 .4 .6 .9
Total ~ 5
5
U VW X Y
.2 .6 .8 1.5 1.9
Total - 5
6
AB C D E
.3 .5 .9 1.35 1.95
Total • 5
7
F G H I J
.05 .5 1.05 1.5 1.9
Total • 5
8
K L M N O
.05 .5 1.05 1.5 1.9
Total • 5
9
P QR S T
.05 .8 1 1.4 1.9
Total ~5.25
10
U V WXY
.4 .95 1.4 1.65 1.8
Total • 6.2 to 6.25

Figure 6:38
Paul's Worksheet F-3

MARKING HOMEWORK

The following worksheet was given to Eric Counter for homework. Mark the work correcting all the mistakes.
Can you see what Eric is doing wrong?


Reading Scales
Name: Eric Counter
2.5
2.75
(a) 0.4
(b) 1.5
(c) 2.4
(d) 2.74
0
1
2
3
2.6
3.6
4.95
6.1
(e) 2.7
(f) 3.3
(g) 4.41
(h) 6.4
3
4
5
6
5.7
6.6
8.05
(i) 6.6
(j) 6.12
(k) 8.1
6
7
8
2.11
2.175
(l) 2.6
(m) 2.2
(n) 2.1.7
2
2·1
2·2
(o) 10.4
(p) 10.6
10
11

interchange decimals and percents, by moving the decimal point two places to the left or right. He also knew how to interchange fractions and decimals. He realized that with any given fraction, he could divide the denominator into the numerator and get the correct decimal equivalent every time.

He also knew that decimals could be written as fractions by writing them with a denominator that was a factor of ten. When he was asked how he knew exactly which denominator to choose, he said that it depended on the number of digits after the decimal point, and replied that "If there's one number after the decimal point, that's tenths, so the denominator would be ten, two numbers would be one hundred."

Paul knew the meaning of the symbols < and > on worksheet G-4 (Figure 6:39). He was the only student who did not ask the teacher/researcher the meaning of the symbols. He did not correctly place all the decimals, fractions, and percents in the proper columns but his errors were few. He correctly placed the symbols =, <, and > in the given spaces.

Operations

Paul had an excellent procedural knowledge base and he was able to perform all operations efficiently. He was able to distinguish which operation to use in a written problem. Paul was not confused by problems dealing with percentages. He sometimes liked to make comments in response to a question (Figure 6:40). He knew how to find the answer to the third question yet he did not complete the solution. When asked why the solution had not been completed he said, "I don't feel like it but I know how to do it." He then explained the procedure to the teacher/researcher. There were very few errors on Paul's operations' worksheets. One error appeared when asked to halve each term in the following sequence:

3.2, 1.6, ___, ___, ___, ___, ___, ___, ___.

His responses were "3.2, 1.6, .8, .4, .2, .1, .5, .25, .125." Having correctly completed the sequence up to the term .1, he then said that the next term was .5 which would be half of one but not half of .1. When he was asked about the halving sequence, he immediately realized that there was an error and that .5 was incorrect. He rapidly corrected that error and the remainder of the sequence without difficulty.

Paul seemed to have found a pattern in "Complete the Pyramid" (Figure 6:41) and he observed that the left and right columns increased by one hundredth, and correctly filled in those spaces with hundredths. It seemed that Paul had noticed that the sum of two adjacent numbers gave the number directly below those two numbers. However, he had not used this rule throughout the pyramid. In row five he added 0.05 and 0.11 to give 0.16, and 0.05 with 0.14 to give 0.19. The response 0.19 was incorrect. Even with this error, the procedure could still be carried through although it

Figure 6:39
Paul's Worksheet G-4

FRACTIONS/DECIMALS/PERCENTS

The following rectangle contains fractions, decimals and percents. Take each number and place it in the correct column in the second rectangle.

$\frac{1}{3}$ $\frac{25}{100}$ $\frac{2}{10}$ $\frac{3}{8}$ 17%
50% 18 100% $\frac{29}{100}$ $\frac{23}{100}$
$\frac{3}{4}-\frac{1}{2}$ $\frac{3}{10}$ .18 25% .09
.25 5% $\frac{5}{8}-\frac{1}{2}$ 2.1 $\frac{25}{10}$ .5 $\frac{1}{5}$

$< \frac{1}{4}$	$\frac{1}{4}$	$> \frac{1}{4}$
2/10 3/8 17% 5% $\frac{23}{100}$ 1/8 1/5 .18 .09	$\frac{25}{100}$ 25% .25 3/4 - 1/2	$\frac{1}{3}$ 100% 50% $\frac{3}{10}$ $\frac{29}{100}$ $\frac{25}{10}$ 2.1 .5

Make the following true, using

= means "equals"
< means "less than"
> means "greater than"

$\frac{1}{2}\ \boxed{=}\ 50\%$ $\frac{1}{4}\ \boxed{<}\ 30\%$ $10\%\ \boxed{=}\ \frac{10}{100}$

$\frac{1}{3}\ \boxed{>}\ 25\%$ $\frac{2}{5}\ \boxed{=}\ 40\%$ $75\%\ \boxed{<}\ .8$

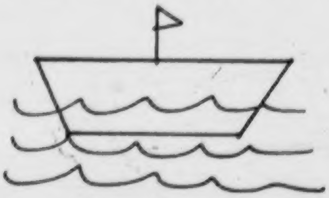
$90\%\ \boxed{=}\ \frac{90}{100}$ $100\%\ \boxed{=}\ \frac{100}{100}$ $100\%\ \boxed{=}\ 1$

$125\%\ \boxed{=}\ 1\frac{1}{4}$ $150\%\ \boxed{<}\ 1\frac{3}{4}$ $200\%\ \boxed{>}\ 1\frac{1}{2}$

$23\%\ \boxed{<}\ \frac{1}{4}$ $\frac{3}{4}\ \boxed{>}\ 72\%$ $1\%\ \boxed{<}\ .1$

Figure 6:40
Paul's Operations Examples

The gunwale of a boat is 2.5 meters above water. If the tide rises 1 meter, how much of the gunwale of the boat is now above water? (From worksheet H-3)



2.5 meters

TRICKY

You took three of your friends to the best restaurant in town for dinner one night. The bill came to $384.79. The service was great and you wish to leave a 15% tip. How much should you leave for a tip ? (From worksheet H-3)

NOTHING, YOU'VE SPENT TOO MUCH ALREADY.

Martha wants to make $55 this week. Her part time job pays $2.75 per hour. How many hours must she work ? (From worksheet H-6)

$2.\underbrace{75}\ \overline{)55.\underbrace{00}}$ LOTS!

would mean that some of the other responses were incorrect. The arrows on Figure 6:41 were placed by the teacher/researcher to indicate Paul's pattern.

Interpretation of Paul's Work

Of the eight students who took part in the teaching experiment, Paul had the most correct procedural knowledge. The pretest interview and the delayed interview confirmed Paul's very good procedural knowledge.

Paul had confidence in his procedural knowledge and his procedures did not need reinforcement. Not once during the teaching experiment did Paul ask the teacher/researcher how to do something nor did he ask for verification concerning a procedure.

Paul showed more maturity than did the other students. He examined each worksheet before completing any responses; he did not need any assistance; and he did not become impatient, frustrated or upset.

Paul could express himself more clearly than the other students. Where other students would say, "It's because this number is bigger (and point to a number)", Paul would say, "It's because the tenths are bigger".

Paul's efficient procedural knowledge placed him at the rim of the procedural layer and the psychological growth layer (level IV). However, his growth stopped here. In Paul's case it seemed possible that the procedural layer could be peeled and the abstract layer attained. He managed to learn and retain the procedures necessary to take effective action when presented with a situation, yet he had not progressed beyond the procedural layer.

A statement was made by Paul during the delayed interview which allowed the teacher/researcher to believe that Paul could attain the abstract layer. During the delayed interview Paul was asked the following question which appeared on the post-test and delayed post-test.

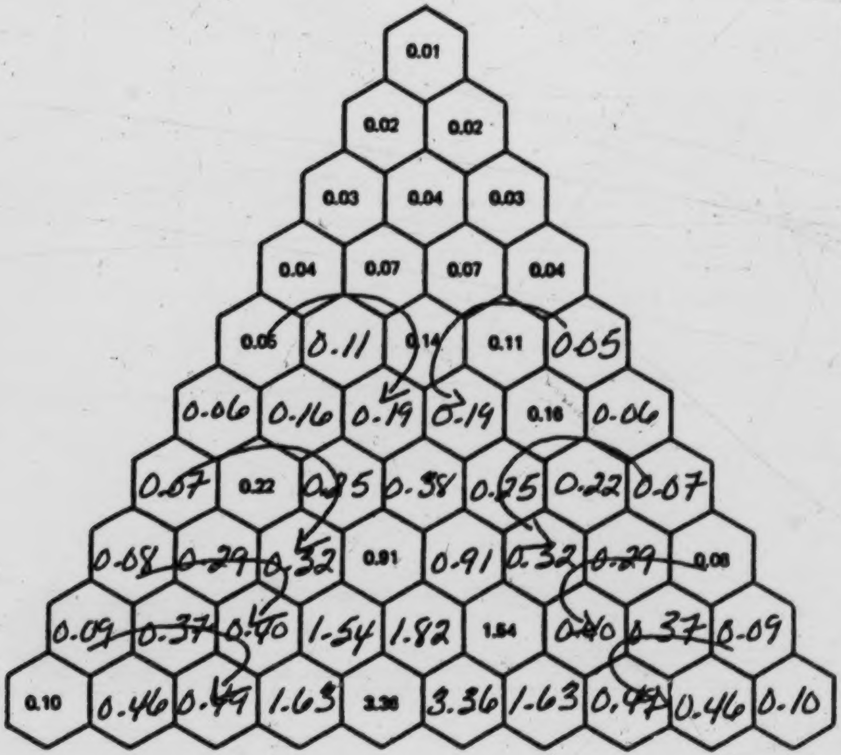
> Muscle makes up 0.4 of the total body mass. Wendy has a mass of 44.8 kg. How many kilograms of muscle does Wendy have?
>
> a) 44.8 - 0.4 b) 44.8 ÷ 0.4
> c) 44.8 + 0.4 d) 44.8 x 0.4
> e) 0.4 ÷ 44.8

Figure 6:41
Paul's Worksheet H-4

COMPLETE THE PYRAMID

Study the following pyramid to see how the numbers are obtained.

Complete the pyramid by supplying the missing numbers.



0.01
0.02 0.02
0.03 0.04 0.03
0.04 0.07 0.07 0.04
0.05 0.11 0.14 0.11 0.05
0.06 0.16 0.19 0.19 0.16 0.06
0.07 0.22 0.25 0.38 0.25 0.22 0.07
0.08 0.29 0.32 0.91 0.91 0.32 0.29 0.08
0.09 0.37 0.40 1.54 1.82 1.54 0.40 0.37 0.09
0.10 0.46 0.49 1.63 3.36 3.36 1.63 0.47 0.46 0.10

His reply was:

> 'I know the answer is (d), multiply, because the answer has to be smaller than 44.8 and when you multiply two decimal numbers the answer is smaller. It sounds as if the answer should be divide but when you divide two decimal numbers the answer is bigger. I don't know why that is. It doesn't make any sense to me, but that's the way it is.'

Because Paul's experiences with the decimal number system did not go beyond the procedural layer, neither did he.

Paul's post-test score (70%) and his delayed post-test score (82%), both of which were significantly above the post-test and delayed post-test means, indicated Paul's knowledge of correct procedures.

PRESENTATION OF CARMEN'S WORK

Carmen did not complete all the worksheets (Figure 6:42). She was always fifteen to twenty minutes late for each class. She did not take up what she missed, but picked up where the group was.

Place Value

Carmen showed difficulty with the place value system (Figure 6:43). She was unable to order or compare decimals (questions 2,3, and 4), and her responses showed no consistency or pattern. She seemed to be randomly choosing responses. Although she circled the correct number in question 1 she could not explain why 10.9 was larger than 10.75. Carmen was not very good at expressing herself verbally. She did not complete E-4 because when she arrived in class the other members of her group had finished E-4 and were on E-5.

While the two other members of her group were working on worksheet E-6, Carmen was still working on E-5. She did not complete worksheet E-6 because the group had finished E-6 while she was still working on E-5.

Carmen took her time completing the worksheets; she sometimes worked so long on one worksheet that her groupmates had time to complete the next worksheet. In this way Carmen also missed the next worksheet because once she had finished the worksheet she had begun, she would rejoin the group at the point where they were. She did not see any need to complete the sheets she had missed.

Carmen was able to add decimal numbers and she simplified the expanded forms of the decimals before representing them on the abacus (Figure 6:44).

Figure 6:42
Carmen's Worksheets

Place Value		Scale Reading		Fraction-to-Decimal Translation		Operations	
E-1		F-1		G-1		H-1	✓
E-2		F-2		G-2		H-2	✓
E-3		F-3		G-3		H-3	A
E-4		F-4		G-4	✓	H-4	✓
E-5	✓					H-5	✓
E-6						H-6	✓
E-7	✓						
E-8	✓						
E-9	✓						
E-10							
E-11							

/ Completed A Attempted C Copied

Figure 6:43
Carmen's Worksheet E-5

DECIMALS

1. CIRCLE the larger of the two numbers:

[✓ 10.9] or 10.75

How do you know it is bigger?

2. Write the following numbers in decreasing order (BIGGEST first).

1.9, 1.86, 1.842, 1.7756, 2.5, 2.1, 2.07

1.86 1.9 2.07 2.5 1.842 2.1 1.7756

3. CIRCLE all the numbers that are BIGGER than 0.4:

[✓ 0.15] 0.09 [✓ 0.5] 0.104 0.3

CIRCLE all the numbers that are SMALLER than 0.45:

[✓ 0.6] 0.125 [✓ 0.373] [✓ 0.3] 0.085

4. Write down any number that is BIGGER than the first, but SMALLER than the second.

example: $6 < 8.5 < 10$

a) 6.2 < 6.3 < 6.5
b) 0.9 < 0.8 < 1
c) 1 < 1.0 < 1.1

5. In each of the following, CIRCLE the two numbers which are equal:

a) [✓ 098] 908 980 [✓ 98]

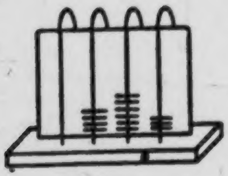
b) [✓ 0.32] 0.032 0.302 [✓ 0.320]

c) [✓ 06.5] 60.5 6.05 [✓ 6.50]

Figure 6:44
Carmen's Worksheet E-7

DECIMALS AND THE ABACUS

46.3 can be shown on an abacus

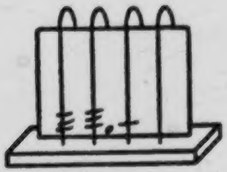


The number 46.3 is written below in expanded form.

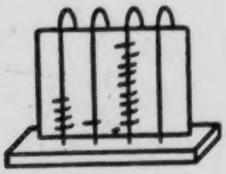
40 + 6 + 0.3

= 4 tens + 6 ones + 3 tenths
= (4x10) + (6x1) + (3x0.1)
= (4x10) + (6x1) + (3x1/10)

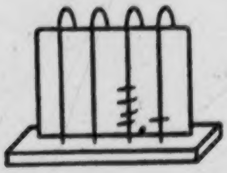
A.
Show each of the following expanded forms on the abacus.



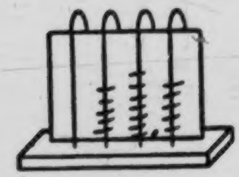
1. 30 + 3 + 0.1



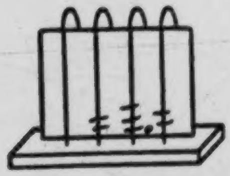
2. 40 + 1 + 0.9



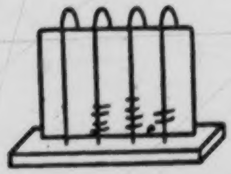
3. 4 + 0.1



4. 50 + 7 + 0.5

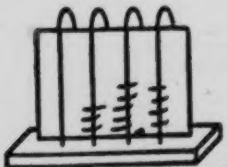


5. (5 x 4) + (3 x 1) + (2 x 0.1)
20 + 8 + .2

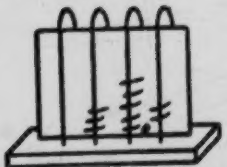


6. (4 x 10) + (4 x 1) + (2 x 0.1)
40 + 4 + .2

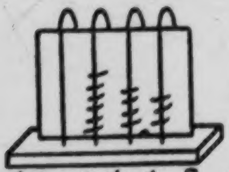
0.1
2
0.2



30 + 5 + .4
7. (3 x 10) + (5 x 1) + (4 x 0.1)



30 + 5 + .2
8. (3 x 10) + (5 x 1) + (2 x 0.1)



60 + 4 + .3
9. (6 x 10) + (4 x 1) + (3 x 0.1)

Carmen liked worksheets E-8 and E-9 because she found them easy. Extracts from these two worksheets were shown in Figure 6:45. Her comment was, "This is easy. I only have to put marks on the lines. I don't have to think."

Carmen's procedural knowledge of the place value system was very limited.

Scale Reading

Carmen was absent when the scale reading worksheets were discussed and she did not take them up later.

Fraction-to-Decimal Translation

Carmen arrived halfway through the class and missed most of the fraction-to-decimal worksheets. She completed only worksheet G-4 (Figure 6:46). She was working with Ashley and Jessica who also had difficulty with fraction-to-decimal translation. Carmen did not know what the symbols < and > meant so she asked the teacher/researcher. She had difficulty comparing fractions, decimals, and percents and relied on the other members of her group to complete this worksheet.

Carmen did not do enough of the fraction-to-decimal translation worksheets to judge her knowledge in this area.

Operations

Carmen completed all the operations' worksheets. Carmen's procedural knowledge about the addition and subtraction of decimals was good. She could not always interpret whether a problem was asking for addition or subtraction. For example, the following question appeared on worksheet H-3:

> Bonnie Jones and her family heated 342.75 gallons of hot water one month. This was 56.47 gallons less than the previous month. How many gallons did they heat in the previous month?

Carmen performed a subtraction rather than an addition because the problem said less. Carmen did not interpret the meaning of the problem but used the words in the problem as clues to an operation.

Carmen also had difficulty with multiplication and division problems. She did not always choose the correct operation and when she did, the operation was not always performed correctly.

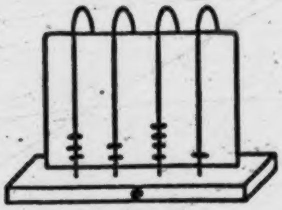
These inconsistencies were noticed on worksheet H-6 (Figure 6:47). Questions 1,3,4, and 7 show a correct operation and a correct procedure although question 4 had

Figure 6:45
(From Worksheet E-8)

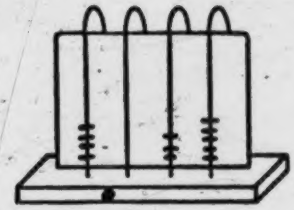
NUMBERS ON THE ABACUS

What number is represented on each abacus?
The first two are done for you.

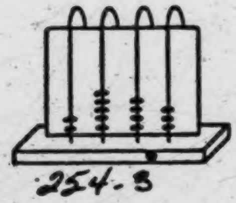
1. 32.41



2. 4.035

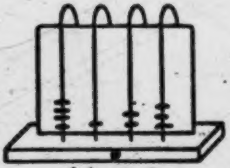


3.



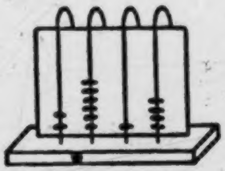
254.3

4.



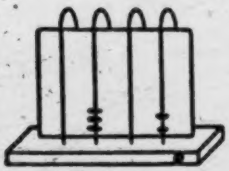
41.23

5.



2.614

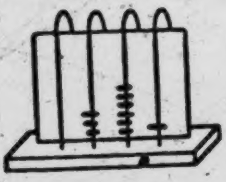
6.



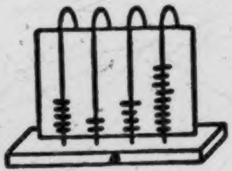
0302.

(From Worksheet E-8)

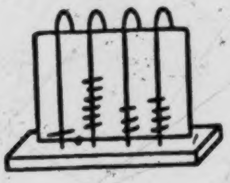
On each abacus, show the numeral written under
it. The first two are done for you.



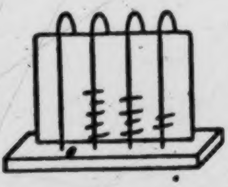
1. 36.1



2. 62.49

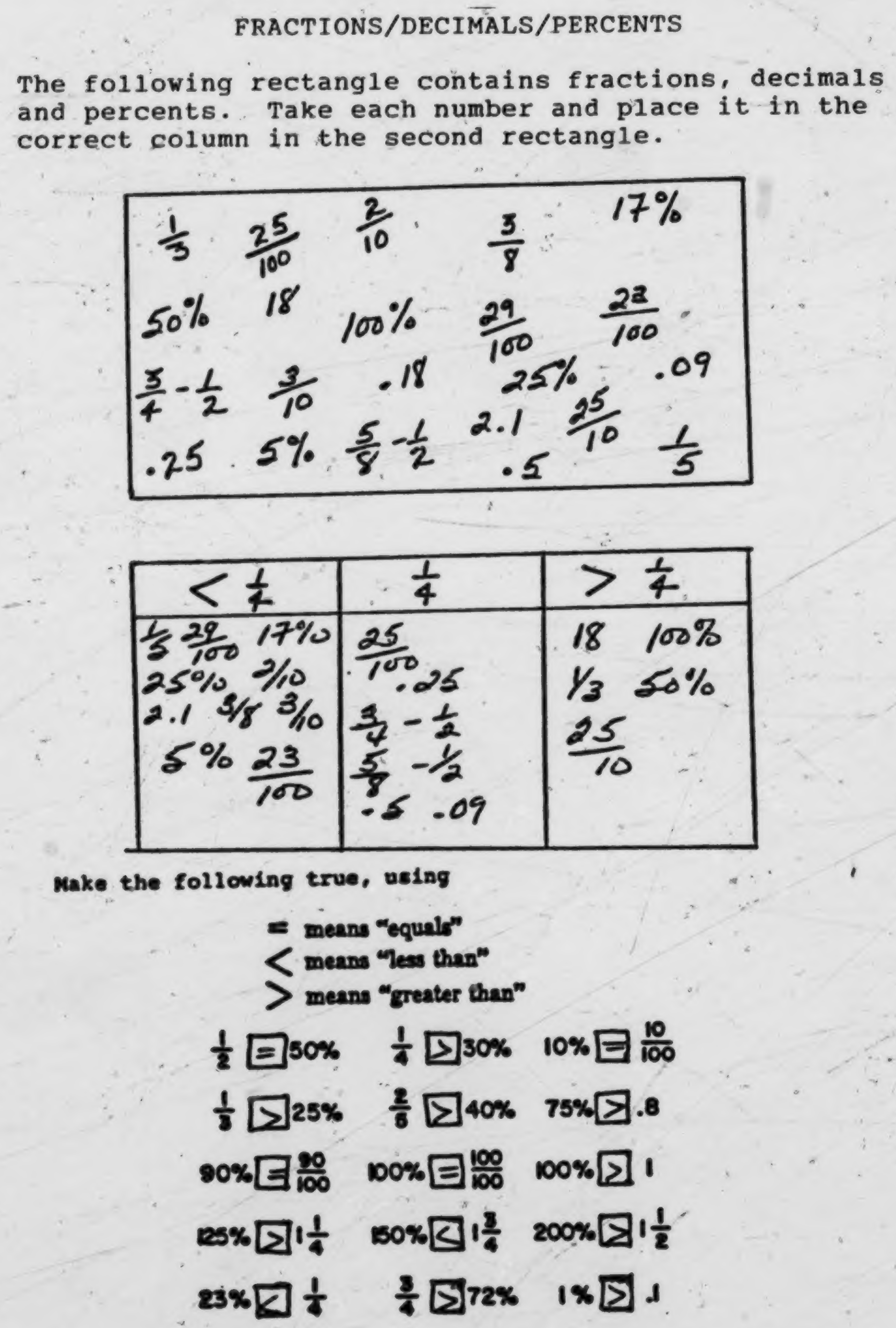


3. 1.634



4. 0.542

Figure 6:46
Carmen's Worksheet G-4



FRACTIONS/DECIMALS/PERCENTS

The following rectangle contains fractions, decimals and percents. Take each number and place it in the correct column in the second rectangle.

$\frac{1}{3}$ $\frac{25}{100}$ $\frac{2}{10}$ $\frac{3}{8}$ 17%
50% 18 100% $\frac{29}{100}$ $\frac{22}{100}$
$\frac{3}{4}-\frac{1}{2}$ $\frac{3}{10}$ .18 25% .09
.25 5% $\frac{5}{8}-\frac{1}{2}$ 2.1 .5 $\frac{25}{10}$ $\frac{1}{5}$

$< \frac{1}{4}$	$\frac{1}{4}$	$> \frac{1}{4}$
$\frac{1}{3}$ $\frac{29}{100}$ 17% 25% 2/10 2.1 3/8 3/10 5% $\frac{23}{100}$	$\frac{25}{100}$ .25 $\frac{3}{4}-\frac{1}{2}$ $\frac{5}{8}-\frac{1}{2}$ .5 .09	18 100% 1/3 50% $\frac{25}{10}$

Make the following true, using

= means "equals"
< means "less than"
> means "greater than"

$\frac{1}{2}\ \boxed{=}\ 50\%$ $\frac{1}{4}\ \boxed{>}\ 30\%$ $10\%\ \boxed{=}\ \frac{10}{100}$

$\frac{1}{3}\ \boxed{>}\ 25\%$ $\frac{2}{5}\ \boxed{>}\ 40\%$ $75\%\ \boxed{>}\ .8$

$90\%\ \boxed{=}\ \frac{90}{100}$ $100\%\ \boxed{=}\ \frac{100}{100}$ $100\%\ \boxed{>}\ 1$

$125\%\ \boxed{>}\ 1\frac{1}{4}$ $150\%\ \boxed{<}\ 1\frac{3}{4}$ $200\%\ \boxed{>}\ 1\frac{1}{2}$

$23\%\ \boxed{<}\ \frac{1}{4}$ $\frac{3}{4}\ \boxed{>}\ 72\%$ $1\%\ \boxed{>}\ .1$

Figure 6:47
Carmen's Worksheet H-6

MULTIPLY AND DIVIDE

1. Jovani made a long-distance call from Antigonish, N.S. to Moncton, N.B., on Sunday at 9 a.m. He talked for 7 minutes. What did the call cost him if the rate was $0.58 per minute?

$$\begin{array}{r} 58 \\ 7 \\ \hline 4.06 \end{array}$$

2. Martha wants to make $55 this week. Her part time job pays $2.75 per hour. How many hours must she work?

$$55\overline{)275}$$

She has to work 5 hours.

3. While hauling a truck load of oranges, the transport truck was involved in an accident and spilled 2604 oranges. How many dozen oranges were spilled?

$$\begin{array}{r} 217 \\ 12\overline{)2604} \\ 24 \\ \hline 2580 \\ 12 \\ \hline 2568 \\ 2556 \end{array}$$

217 dozen oranges were spilt.

4. For a championship football game between the Wildcats and the Reddogs the admission charge was $2.25. The price included the bus ride to and from the stadium. How much did it cost 347 people to attend?

$$\begin{array}{r} 2.25 \\ 347 \\ \hline 1575 \\ 1000 \\ 57500 \\ \hline 600\ 75 \end{array}$$

It will cost $600.75 for 347 people to attend.

5. A football stadium has seating for 68300 people. If 84% of the seats are occupied, how many people are at the football game?

$$\begin{array}{r} 800 \\ 84\overline{)68300} \end{array}$$

800 people are at the football game.

Figure 6:7 cont'd

6. A class of 18 Math 15 students is planning a field trip to New York. The total cost of the trip is $5760.00. If the school gives $900 towards the cost of the trip, how much will each student have to pay to go on the trip?

5760.00
9.00
5751.00

Each student will have to pay $57.51 in all.

7. An apple has 28.4 mg of vitamin C. A banana has 11.6 mg of vitamin C.

a) How much more vitamin C is there in an apple than in a banana ?

7 1
28.4
11.6
16.8

There is 16.8 mg more in an apple.

b) How much vitamin C would you get if you ate 2 bananas and half an apple ?

1
11.6
2
23.2

the wrong answer. Question 2 had a correct operation but an incorrect procedure. Carmen had performed a division but had not performed the division correctly; she did not see that her answer was unreasonable. Her computation gave her a whole number response which she assumed was correct. Carmen liked whole number answers and having calculated one, she felt confident it was the correct response. An incorrect operation was performed for question 5. She knew that the answer was less than 68 300, so she divided. Question 6 was not completed to the end. Two operations were required, subtraction and then division. Carmen completed the subtraction (incorrectly) and stopped there. Because she was unable to recognize whether her answer was reasonable or not she therefore, saw no need for a second operation in order to complete the problem. Her only concern during the teaching experiment was to do the problem. Once a problem was solved there was no going back.

It came as a complete surprise to her groupmates when Carmen found the pattern and completed the pyramid (Figure 6:48) without the aid of the teacher/researcher or her group members. The two members of her group looked at the page and said, "How do you do this?" Carmen replied, "It's easy. Don't you see what's happening?" Her two groupmates looked at her with surprise because until that point, they had helped her with the other worksheets and she had contributed very little. Carmen said, "You add two numbers to make up the one underneath." She then used her diagram to show them what she meant. Carmen's only error was one term in the last row.

Interpretation of Carmen's Work

Carmen was always fifteen to twenty minutes late for class. She noticed this herself and made the comment, "I'm sorry I'm late. I always seem to be late." Because of her tardiness, she did not complete a number of the worksheets. When she did arrive in class, she did not take up any worksheets she had missed but she would begin at the point where her group was.

Carmen was very concerned about the procedures and about getting the correct answer. She was very unsure of her procedures and required constant reinforcement from the teacher/researcher. Before beginning a worksheet she would ask the teacher/researcher, "How do you do this?", and before completing the worksheet she would ask, "Is this right?" Because she was concerned about the procedures and the correct answer, she worked more slowly than Ashley and Jessica, the other members of her group, who would move on to the next worksheet. When this happened Carmen would often skip a worksheet to catch up to them.

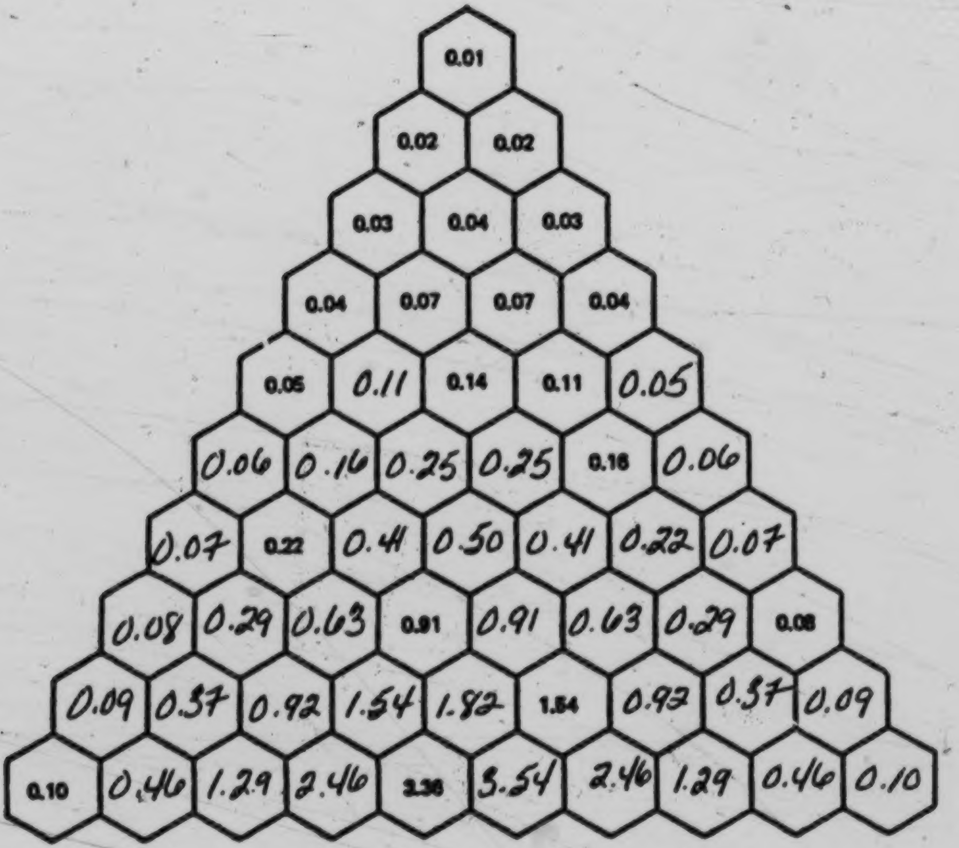
Carmen made a comment which led the teacher/researcher to believe that Carmen saw the procedure and the understanding as the same thing. She said, "I want you to tell me how to do this. I want to understand." If she could calculate the solution, then that was all she wanted to know and that was all that was important or necessary to her. Knowing how to calculate the solution was what she needed to know for her to function

Figure 6:48
Carmen's Worksheet H-4

COMPLETE THE PYRAMID

Study the following pyramid to see how the numbers are obtained.

Complete the pyramid by supplying the missing numbers.



0.01
0.02 0.02
0.03 0.04 0.03
0.04 0.07 0.07 0.04
0.05 0.11 0.14 0.11 0.05
0.06 0.16 0.25 0.25 0.16 0.06
0.07 0.22 0.41 0.50 0.41 0.22 0.07
0.08 0.29 0.63 0.91 0.91 0.63 0.29 0.08
0.09 0.37 0.92 1.54 1.82 1.54 0.92 0.37 0.09
0.10 0.46 1.29 2.46 3.36 3.54 2.46 1.29 0.46 0.10

within the system of decimal numbers.

Carmen was very pleasant, polite and calm and she did not become frustrated, impatient or upset. She always checked her procedures and responses with the teacher/researcher, and required constant reassurance.

Carmen's procedural knowledge was poor but she knew just enough about correct procedures to function and get by within the decimal number system. Carmen could be placed at level II of the procedural layer. She would not likely escape from the procedural layer. Her insistence on the procedures as a way of understanding would not permit her to attain the abstract layer. These procedures need reinforcing but she will always function at the procedural layer.

Carmen displayed poor procedural knowledge; she was not able to take effective action when a situation was presented to her. Her post-test score (32%) and her delayed post-test score (27%), which were both below the post-test and delayed post-test mean, indicate her lack of correct procedures. The teaching experiment did not seem to remedy any of her incorrect procedures.

PRESENTATION OF JESSICA'S WORK

Jessica did not complete all of the worksheets (Figure 6:49). She did not take up the worksheets she missed on the days she was absent and would begin at the point where her group was at.

Place Value

Jessica completed five of the eleven place value worksheets. She began with worksheet E-6 (Figure 6:50) and completed this worksheet rather well; it contained two errors, on items 2a and 2b where she changed the tenths instead of the hundredths. When speaking to Jessica, she was able to specify the place value positions of the digits in a decimal number.

Her work on decimals and the abacus (Figure 6:51) was not as well done. Questions 1 to 4 were correct and questions 5 to 9 were not. She was able to represent the given expanded forms of decimal numbers on the abacus by mentally adding the terms. She used the same procedure for questions 5 to 9 but in a slightly different way. For example, the expanded form (5 x 4) + (3 x 1) + (2 x 0.1) was simplified to 25.1 rather than 23.2. She took the sum of 5 x 4 and 3 x 1, 23, to which she added 2 and then .1 to make 25.1. She used this process where she should have simplified the brackets and then summed the separate terms.

Jessica and Ashley completed worksheet E-10 (The Fewest Number of Coins)

Figure 6:49
Jessica's Worksheets

Place Value		Scale Reading		Fraction-to-Decimal Translation		Operations	
E-1		F-1	✓	G-1		H-1	✓
E-2		F-2	✓	G-2		H-2	A
E-3		F-3	✓	G-3		H-3	
E-4		F-4		G-4	✓	H-4	✓
E-5						H-5	
E-6	✓					H-6	
E-7	✓						
E-8	✓						
E-9	✓						
E-10	✓						
E-11							

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Figure 6:50
Jessica's Worksheet E-6

1. Write the number that is one tenth greater.

a) 2.5 2.6 b) 1.9 2.0
c) 25.7 25.8 d) 0.68 0.69
e) 125.2 125.3 f) 0.7 0.8

2. Write the number that is one hundredth greater.

a) 8.75 8.85 b) 0.93 0.103
c) 1.085 1.095 d) 3.346 3.356
e) 17.591 17.601 f) 0.083 0.093

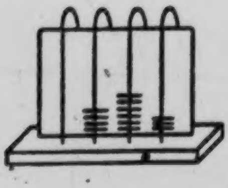
3. Write the number that is one thousandth less.

a) 5.238 5.237 b) 6.088 6.087
c) 0.13 0.129 d) 4.959 4.958
e) 6.781 6.780 f) 15.326 15.324

Figure 6:51
Jessica's Worksheet E-7

DECIMALS AND THE ABACUS

46.3 can be shown on an abacus

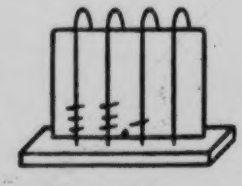


The number 46.3 is written below in expanded form.

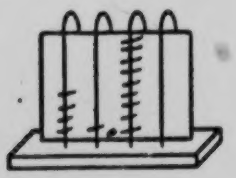
40 + 6 + 0.3

= 4 tens + 6 ones + 3 tenths
= (4x10) + (6x1) + (3x0.1)
= (4x10) + (6x1) + (3x1/10)

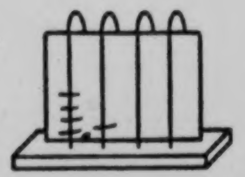
A.
Show each of the following expanded forms on the abacus.



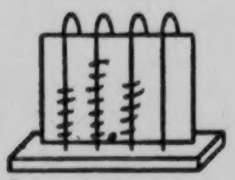
1. 30 + 3 + 0.1



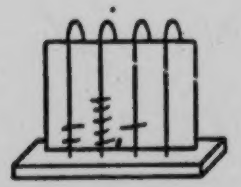
2. 40 + 1 + 0.9



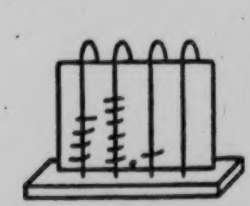
3. 4 + 0.1



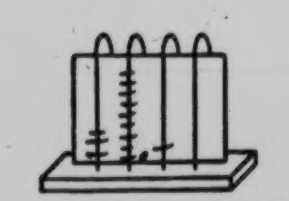
4. 60 + 7 + 0.5



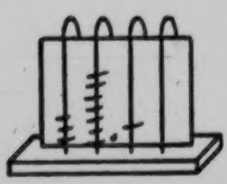
5. (5 x 4) + (3 x 1) + (2 x 0.1)



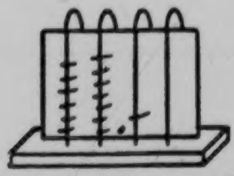
6. (4 x 10) + (4 x 1) + (2 x 0.1)



7. (3 x 10) + (5 x 1) + (4 x 0.1)



8. (3 x 10) + (5 x 1) + (2 x $\frac{1}{10}$)



9. (6 x 10) + (4 x 1) + (3 x $\frac{1}{10}$)

together. They were to record the fewest number of coins or bills needed to make a given amount. They took turns completing a row and they said that it was easy. This served as an intermission between worksheets.

Jessica did not complete enough of the place value worksheets to get a good idea of her procedures and processes.

Scale Reading

Jessica displayed good scale reading abilities. She was able to read scales subdivided into units other than tenths (Figure 6:52). She correctly read the scales which were subdivided into tenths (questions 1,3,6,7). The scales which were subdivided into fifths (questions 2 and 5) were cut into tenths before a response was attempted. Jessica could correctly read scales to the hundredths place (questions 6,7,8,9,10). Her scale reading abilities were better than the abilities of the other students. The teacher/researcher would have liked Jessica to complete another scale reading worksheet to see whether her scale reading ability was consistent; however, she completed only one scale reading worksheet. After she had finished this sheet she did not want to attempt any more worksheets.

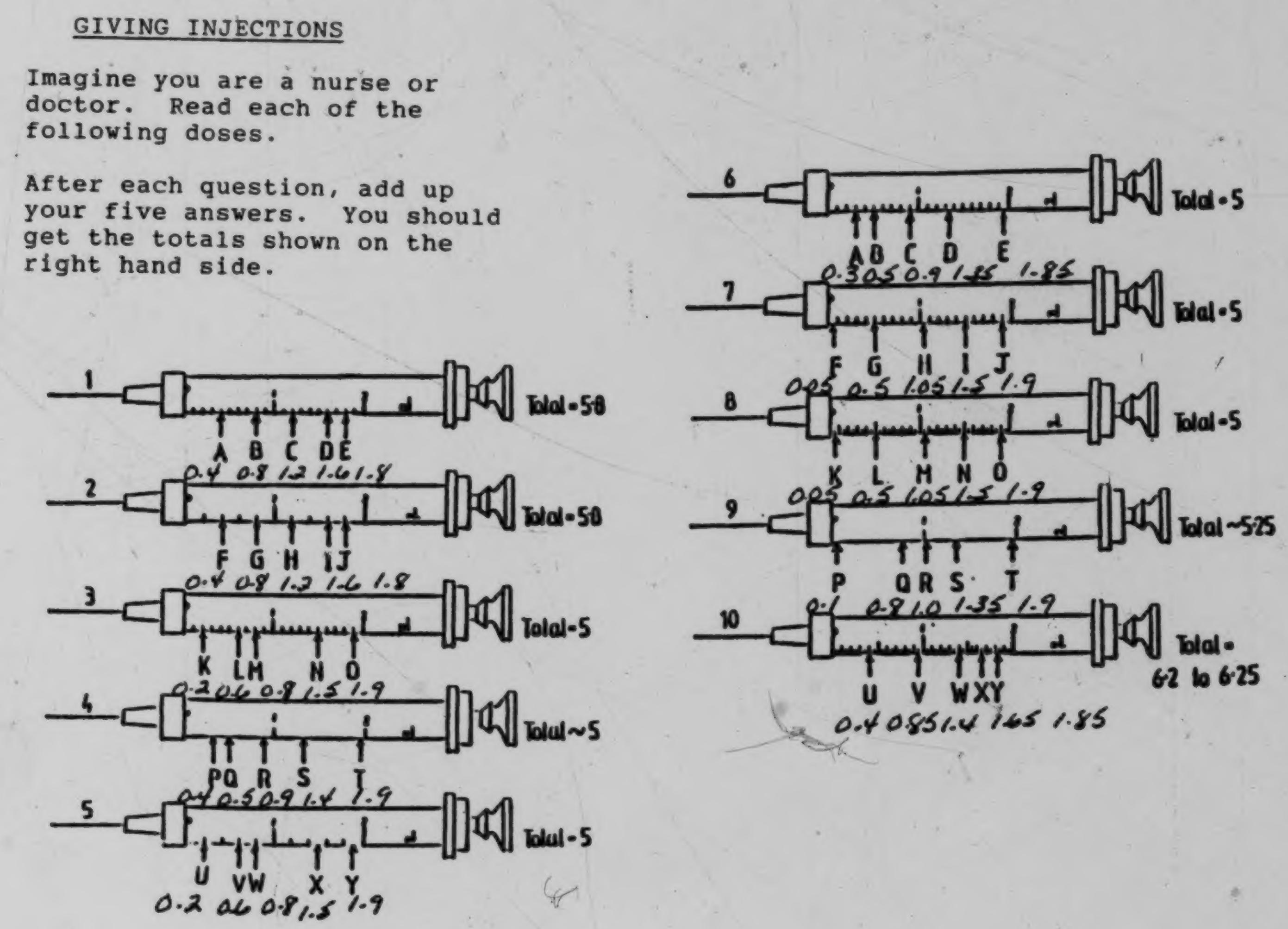
Fraction-to-Decimal Translation

Jessica completed only one fraction-to-decimal translation worksheet (Figure 6:53). She had difficulty with this worksheet, as did most of the other students. To begin with, she did not know what the symbols < and > meant. She asked the teacher/researcher what they meant. She had difficulty comparing various fractions, decimals and percents to the fraction one quarter. This worksheet appeared to present many difficulties for the students. It seemed as if it were too much all at once. Students had to deal with fractions, decimals, percents and then compare each one to the given fraction one fourth. Jessica worked out a few of these and became frustrated with the fraction one third. She asked the teacher/researcher if one third was bigger than one fourth. She was unable to find a way of comparing those two fractions. The teacher/researcher asked her to draw two circles and to cut one circle into fourths and the other circle into thirds. She was then asked how could she compare one third and one fourth using these two diagrams. She shaded one part in each diagram and saw that one third was indeed bigger than one fourth.

These Math 15 students were unable to think of a way to compare fractions, decimals and percents.

It is difficult to access Jessica's knowledge of fraction-to-decimal translation because she completed only one worksheet.

Figure 6:52
Jessica's Worksheet F-2



GIVING INJECTIONS

Imagine you are a nurse or doctor. Read each of the following doses.

After each question, add up your five answers. You should get the totals shown on the right hand side.

1
A B C DE
0.4 0.8 1.2 1.6 1.8
Total = 5.8
2
F G H IJ
0.4 0.8 1.2 1.6 1.8
Total = 5.0
3
K LM N O
0.2 0.6 0.8 1.5 1.9
Total - 5
4
PQ R S T
0.4 0.5 0.9 1.4 1.9
Total ~ 5
5
U VW X Y
0.2 0.6 0.8 1.5 1.9
Total - 5
6
A B C D E
0.3 0.5 0.9 1.35 1.85
Total = 5
7
F G H I J
0.05 0.5 1.05 1.5 1.9
Total = 5
8
K L M N O
0.05 0.5 1.05 1.5 1.9
Total = 5
9
P QR S T
0.1 0.8 1.0 1.35 1.9
Total ~ 5.25
10
U V WXY
0.4 0.85 1.4 1.65 1.85
Total = 6.2 to 6.25

Figure 6:53
Jessica's Worksheet G-4

FRACTIONS/DECIMALS/PERCENTS

The following rectangle contains fractions, decimals and percents. Take each number and place it in the correct column in the second rectangle.

$\frac{1}{3}$ $\frac{25}{100}$ $\frac{2}{10}$ $\frac{3}{8}$ 17%
50% 18 100% $\frac{29}{100}$ $\frac{23}{100}$
$\frac{3}{4}-\frac{1}{2}$ $\frac{3}{10}$ .18 25% .09
.25 5% $\frac{5}{8}-\frac{1}{2}$ 2.1 $\frac{25}{10}$ .5 $\frac{1}{5}$

less $< \frac{1}{4}$	$\frac{1}{4}$	greater $> \frac{1}{4}$
$\frac{23}{100}$ 1/5 2.1 17% .18 2.5 5% 0.09 .5 $\frac{2}{10}$	$\frac{25}{100}$ 25% .25 5/8 - 1/2	18 1/3 100% 50% 3/10 3/8 29/100

Make the following true, using

= means "equals"
< means "less than"
> means "greater than"

$\frac{1}{2}\ \boxed{=}\ 50\%$ $\frac{1}{4}\ \boxed{<}\ 30\%$ $10\%\ \boxed{=}\ \frac{10}{100}$

$\frac{1}{3}\ \boxed{=}\ 25\%$ $\frac{2}{5}\ \boxed{>}\ 40\%$ $75\%\ \boxed{<}\ .8$

$90\%\ \boxed{=}\ \frac{90}{100}$ $100\%\ \boxed{=}\ \frac{100}{100}$ $100\%\ \boxed{>}\ 1$

$125\%\ \boxed{>}\ 1\frac{1}{4}$ $150\%\ \boxed{=}\ 1\frac{3}{4}$ $200\%\ \boxed{<}\ 1\frac{1}{2}$

$23\%\ \boxed{<}\ \frac{1}{4}$ $\frac{3}{4}\ \boxed{>}\ 72\%$ $1\%\ \boxed{=}\ .1$

Operations

Jessica only completed three of the six operations worksheets. When she felt that she had done enough and did not feel like doing any more she would stop. Worksheet H-1 (Figure 6:54) contained incorrect calculations. In questions 2,3 and 5 she added the wrong term. When this was brought to her attention and she saw her error she said, "I don't feel like changing it", and she did not. Her additions were correct with the term she used. Worksheet H-2 (Figure 6:55) was a worksheet that she did not feel like completing.

Jessica was not able to complete the pyramid without Carmen's help. Carmen told her what the pattern was and made small triangles on Jessica's paper to show her how the sum of two numbers made a third number on the pyramid.

It was difficult to assess Jessica's procedures and processes as she did not complete all the worksheets.

Interpretation of Jessica's Work

Jessica took part in the teaching experiment mostly to get away from the normal classroom routine and to be able to socialize more freely with her friends. Jessica and her groupmates spent a lot of time doing just that. The teacher/researcher would sometimes pass their group and say, "What are you working on?" This would put them on task again.

Jessica decided for herself when she wanted to work and when she did not. Once Jessica had decided that she had done enough for that class she would not look at another worksheet or contribute to the discussion. In addition, she liked to have the teacher/researcher sit next to her. It seemed as if the close proximity of the teacher/researcher was reassuring.

Jessica was working at the procedural layer. She did not complete enough of the worksheets to assess fully her procedural knowledge during the teaching experiment.

Jessica could be placed at level II of the procedural layer. She seemed to be concerned only about working the problem out and at times she was not even interested in that. Her lack of motivation and responsibility make it difficult for her to go beyond the procedural layer.

Jessica's post-test score (38%) indicated a lack of correct procedures. However, her delayed post-test score (53%) showed an increase in correct procedures.

Figure 6:54
Jessica's Worksheet H-1

SEQUENCES

Complete the following sequences:

1. Adding on 0.2's.

0.2, 0.4, 0.6, 0.8 , 1.0 , 1.2 , 1.4 , 1.6 , 1.8 , 2.0 ,
2.2 2.4 , 2.6 ,

2. Adding on 0.3's.

0.3, 0.6, 0.9, 1.2 , 1.4 , 1.6 , 1.8 , 2.0 , 2.2 , 2.4 ,
2.6 , 2.8 , 3.0 ,

3. Adding on 0.4's.

0.4, 0.8, 1.2 , 1.6 , 2.0 , 2.4 , 2.8 , 3.2 , 3.6 , 4.0 ,
4.4 .

4. Adding on 0.5's.

0.5, 1.0 , 1.5 , 2.0 , 2.5 , 3.0 .

5. Adding on 0.25's.

0.25, 0.50 , 1.00 , 1.50 , 2.00 , 2.50 , 3.00 , 3.50 .

6. Adding on 0.05's.

0.05, 0.10 , 0.15 , 0.20 0.25 , 0.30 , 0.35 , 0.40 , 0.45 ,
0.50 , 0.55 , 1.00 , 1.50 , 2.00 , 2.50 , 3.00 , 3.50 , 4.00 , 4.50 .

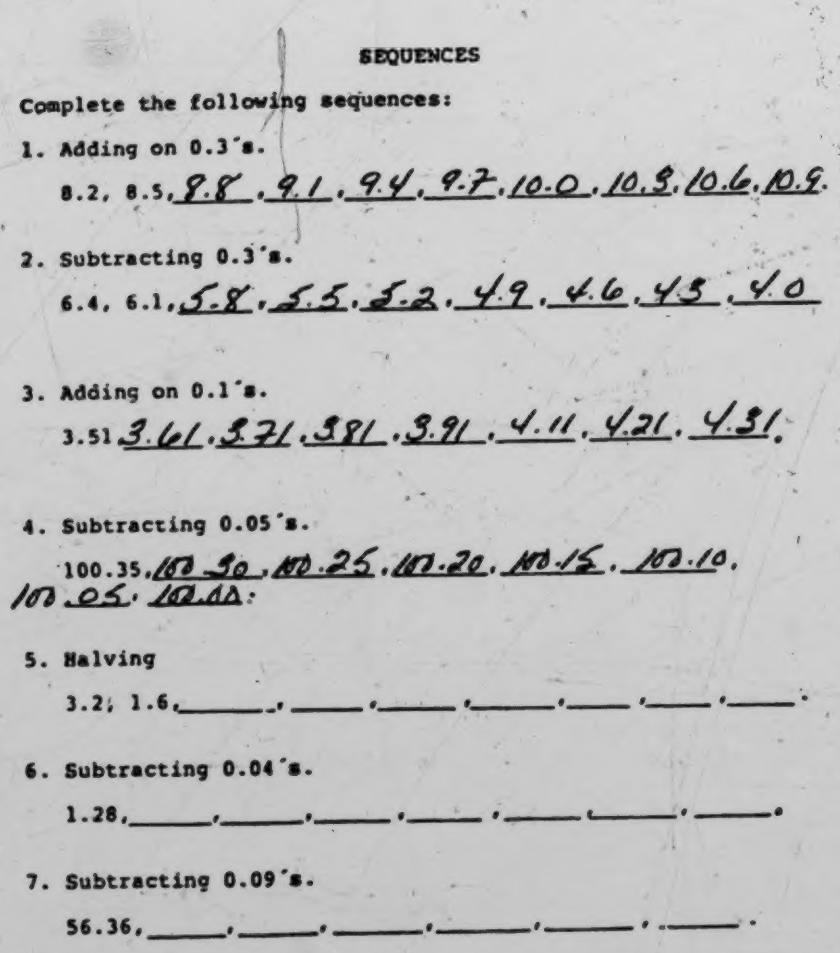
7. Adding on 0.15's.

0.15 0.30 , 0.45 , 0.60 , 0.75 , 0.90 , 1.05 , 1.20 , 1.35 ,
1.50 , 1.65 , 1.80

8. Adding on 0.125

0.125 0.250 0.375 , 0.500 , 0.625 , 0.750 , 0.975 , 1.100 ,
1.225 , 1.350 , 1.475 , 1.600 , 1.725 , 1.850 , 1.975 .

Figure 6:55
Jessica's Worksheet H-2



SEQUENCES

Complete the following sequences:

1. Adding on 0.3's.

8.2, 8.5, 8.8, 9.1, 9.4, 9.7, 10.0, 10.3, 10.6, 10.9.

2. Subtracting 0.3's.

6.4, 6.1, 5.8, 5.5, 5.2, 4.9, 4.6, 4.3, 4.0

3. Adding on 0.1's.

3.51 3.61, 3.71, 3.81, 3.91, 4.11, 4.21, 4.31.

4. Subtracting 0.05's.

100.35, 100.30, 100.25, 100.20, 100.15, 100.10, 100.05, 100.00:

5. Halving

3.2, 1.6, ______, ______, ______, ______, ______, ______, ______.

6. Subtracting 0.04's.

1.28, ______, ______, ______, ______, ______, ______, ______.

7. Subtracting 0.09's.

56.36, ______, ______, ______, ______, ______, ______.

PRESENTATION OF ASHLEY'S WORK

Ashley completed most of the worksheets (Figure 6:56). She was one of the students who was present and who was always on time.

Place Value

Ashley's ability to compare and order decimal numbers improved during the teaching experiment. She began the teaching experiment by being unable to specify place value positions. She stated that 5.75 was the largest of the given numbers in question 1 (Figure 6:57) because it was "closer to the decimal point." This may imply that she was referring to the number in the tenths position. However, if Ashley knew her place value positions to order decimals, she did not use this knowledge in question 2 (Figure 6:57) which had been ordered incorrectly.

Between worksheet E-4 and E-5, Ashley found a way to compare decimals. She stated that 10.9 was larger than 10.75 (question 1, Figure 6:58) because "they are like $10,90." By comparing decimals to money she was able to compare decimal numbers. She thought of 10.9 as $10.90 and 10.75 as $10.75. Whereas question 2 on worksheet E-4 (figure 6:57) was incorrect, a similar question 2 on worksheet E-5 (Figure 6:58) was correct using this new method she had discovered.

During the teaching experiment Ryan expressed difficulty with the comparison of decimal numbers. Ashley said, "Just think of it as money." She was then asked by the teacher/researcher if that rule would work all the time.

A: Yes
T/R: How would it work with 0.37 and 0.3?
A: Add a zero to the three to make thirty cents.
T/R: How would it work with 0.373 and 0.3?
A: Like I said, add a zero to the three to make thirty cents.
T/R: What happens to 0.373?
A: That's thirty-seven cents. You can forget the last number.

Ashley knew that she could add zeros to the end of a decimal number and not change the value of the number. Her procedure worked well enough for her to be satisfied. Ashley did well on worksheet E-6 (Figure 6:59). She correctly identified the tenths and thousandths and all the items dealing with hundredths except two, 2a and 2b.

Ashley's representation of the expanded form of a decimal on an abacus did not match her simplification of the expanded form. Questions 1 to 5 (Figure 6:60) were done correctly but questions 6 to 9 were not. Ashley had simplified each expanded form to a decimal number but represented a different number on the abacus. For example, she simplified the expanded form of (4 x 10) + (4 x 1) + (2 x 0.1) into 44.02 but

Figure 6:56
Ashley's Worksheets

Place Value		Scale Reading		Fraction-to-Decimal Translation		Operations	
E-1	✓	F-1		G-1	A/C	H-1	
E-2	✓	F-2		G-2	A/C	H-2	
E-3	✓	F-3	C	G-3	A/C	H-3	
E-4	✓	F-4		G-4	✓	H-4	✓
E-5	✓					H-5	✓
E-6	✓					H-6	✓
E-7	✓						
E-8	✓						
E-9	✓						
E-10	✓						
E-11							

/ Completed A Attempted C Copied

Figure 6:57
Ashley's Worksheet E-4

COMPARING DECIMALS

1. CIRCLE the largest of these three numbers:

[✓ 5.75] 5.485 5.5

How did you decide that it was the largest?

It's closer to the decimal point.

2. Write the following numbers in ascending order (SMALLEST first).

4	2	5	6	3	7	1
3.87	3.9	4.07	4.5	3.49	3.785	3.6

3. In each list CIRCLE the two numbers that are equal:

a) [✓ 78] 780 [✓ 078] 708

b) [✓ 0.150] [✓ 0.15] 0.015 0.105

4. CIRCLE all the numbers that are BIGGER than 0.6:

0.12 [✓ 0.8] 0.45 0.536 0.4

CIRCLE all the numbers that are SMALLER than 0.75:

[✓ 0.5] [✓ 0.125] [✓ 0.096] 0.8 [✓ 0.106]

5. Write down, if possible, any number that is BIGGER than the first, but SMALLER than the second:

a) 3.5 < 3.6 < 3.7
b) 3.9 < 3.10 < 3.11
c) 4.1 < 4.15 < 4.2
d) 5 < 5.05 < 5.1
e) 5.1 < 5.105 < 5.11

Figure 6:58
Ashley's Worksheet E-5

DECIMALS

1. CIRCLE the larger of the two numbers:

[✓ 10.9] or 10.75

How do you know it is bigger?

Because they are like 8 10.90.

2. Write the following numbers in decreasing order (BIGGEST first).

5	4	3	1	6	2	7
1.86	1.9	2.07	2.5	1.842	2.1	1.7756

3. CIRCLE all the numbers that are BIGGER than 0.4:

0.15 0.09 [✓ 0.5] 0.104 0.3

CIRCLE all the numbers that are SMALLER than 0.45:

0.6 [✓ 0.125] [✓ 0.373] [✓ 0.3] [✓ 0.085]

4. Write down any number that is BIGGER than the first, but SMALLER than the second.

example: 6 < 8.5 < 10

a) 6.2 < 6.3 < 6.5
b) 0.9 < 0.95 < 1
c) 1 < 1.05 < 1.1

5. In each of the following, CIRCLE the two numbers which are equal:

a) [✓ 098] 908 980 [✓ 98]

b) [✓ 0.32] 0.032 0.302 [✓ 0.320]

c) [✓ 06.5] 60.5 6.05 [✓ 6.50]

Figure 6:59
Ashley's Worksheet E-6

1. Write the number that is one tenth greater.

a) 2.5 2.6 b) 1.9 2.0

c) 25.7 25.8 d) 0.68 0.69

e) 125.2 125. 3 f) 0.7 0.8

2. Write the number that is one hundredth greater.

a) 8.75 8.85 b) 0.93 0.103

c) 1.085 1.095 d) 3.346 3. 356

e) 17.591 17.601 f) 0.083 0.093

3. Write the number that is one thousandth less.

a) 5.238 5.237 b) 6.088 6.087

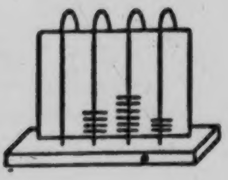
c) 0.13 0. 129 d) 4.959 4. 958

e) 6.781 6. 780 f) 15.326 15.324

Figure 6:60
Ashley's Worksheet E-7

DECIMALS AND THE ABACUS

46.3 can be shown on an abacus

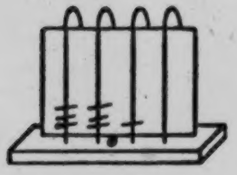


The number 46.3 is written below in expanded form.

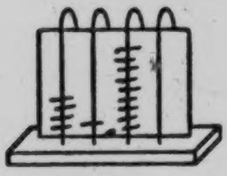
40 + 6 + 0.3

= 4 tens + 6 ones + 3 tenths
= (4x10) + (6x1) + (3x0.1)
= (4x10) + (6x1) + (3x1/10)

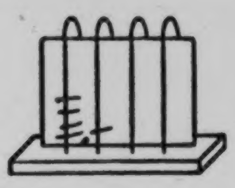
A.
Show each of the following expanded forms on the abacus.



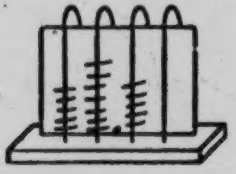
1. 30 + 3 + 0.1



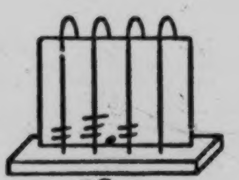
2. 40 + 1 + 0.8



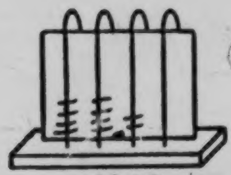
3. 4 + 0.1



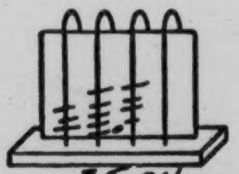
4. 50 + 7 + 0.5



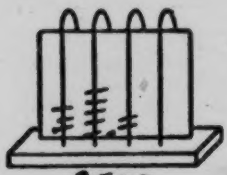
23.2
5. (5 x 4) + (3 x 1) + (2 x 0.1)



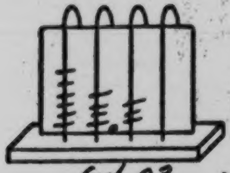
44.02
6. (4 x 10) + (4 x 1) + (2 x 0.1)



35.04
7. (3 x 10) + (5 x 1) + (4 x 0.1)



35.02
8. (3 x 10) + (5 x 1) + (2 x [illegible])



64.03
9. (6 x 10) + (4 x 1) + (3 x [illegible])

represented 44.2 (the correct response) on the abacus. For items 6 to 9, she added the separate terms but placed a zero in the tenths place and the number that should have been in the tenths place was entered in the hundredths place. Yet this is not the number represented on the abacus.

Scale Reading

Ashley had poor scale reading abilities. She did not complete any of the scale reading worksheets. She did fill in a copy of worksheet F-2 (Giving Injections) but the responses were not hers.

Fraction-to-Decimal Translation

Ashley completed the fraction-to-decimal translation worksheets with the help of a groupmate who had very good procedural knowledge.

Ashley, Jessica and Carmen had difficulty comparing fractions, decimals and percents. Worksheet G-4 (Figure 6:61) proved to be extremely difficult. All three members of the group did not know the meaning of the symbols < and >. They asked the teacher/researcher and wrote the information down on their worksheet. Ashley did not have any reference base for comparing fractions, decimals and percents. The three members of this group tried to take one term at a time from the rectangle at the top of the worksheet and attempted to decide where that term fit into the second rectangle. If they did not know or were not sure, they guessed. Once a term had been placed in the second rectangle it was marked off in the first rectangle. This worksheet seemed to have been overwhelming for this group as well.

Operations

Ashley did not complete all the operations' worksheets. She was absent one day and did not take up the missed worksheets.

Ashley's addition and subtraction skills were more developed than her multiplication and division skills. She correctly chose addition and subtraction operations in written problems but multiplication and division proved to be more difficult. Figure 6:62 demonstrated some of Ashley's difficulties. Her only correct response was to question 1 where she chose the correct operation and performed the correct procedure properly. She chose the correct operation for questions 2,3, and 7. However, in question 2 she performed an incorrect procedure. She divided the smaller number into the larger number which did not give the correct response. Once the operation was decided upon, she looked at the numbers to decide how to divide them. She did not consider the question to decide how to divide the numbers to get an answer that would be reasonable. In question 3, Ashley used decimals where there were no decimals to be used. Her procedure was correct except for the use of the decimal

Figure 6:61
Ashley's Worksheet G-4

FRACTIONS/DECIMALS/PERCENTS

The following rectangle contains fractions, decimals and percents. Take each number and place it in the correct column in the second rectangle.

$\frac{1}{3}$ $\frac{25}{100}$ $\frac{2}{10}$ $\frac{3}{8}$ 17%
50% 18 100% $\frac{29}{100}$ $\frac{23}{100}$
$\frac{3}{4}-\frac{1}{2}$ $\frac{3}{10}$ .18 25% .09
.25 5% $\frac{5}{8}-\frac{1}{2}$ 2.1 $\frac{25}{10}$ .5 $\frac{1}{5}$

$< \frac{1}{4}$	$\frac{1}{4}$	$> \frac{1}{4}$
1/5 0.9 17% .18 0.5 23/100 2.1	$\frac{25}{100}$ 100% .25 3/4 − 1/2 5/8 − 1/2	5% 1/3 18 $\frac{29}{100}$ 50% 2/10 3/8 3/10 25/10

Make the following true, using

= means "equals"
< means "less than"
> means "greater than"

$\frac{1}{2}$ $\boxed{=}$ 50% $\frac{1}{4}$ $\boxed{<}$ 30%   10% $\boxed{=}$ $\frac{10}{100}$

$\frac{1}{3}$ $\boxed{=}$ 25% $\frac{2}{5}$ $\boxed{<}$ 40%   75% $\boxed{<}$ .8

90% $\boxed{=}$ $\frac{90}{100}$ 100% $\boxed{=}$ $\frac{100}{100}$ 100% $\boxed{=}$ 1

125% $\boxed{>}$ $1\frac{1}{4}$ 150% $\boxed{=}$ $1\frac{3}{4}$ 200% $\boxed{=}$ $1\frac{1}{2}$

23% $\boxed{>}$ $\frac{1}{4}$ $\frac{3}{4}$ $\boxed{<}$ 72%   1% $\boxed{=}$ .1

Figure 6:62
Ashley's Worksheet H-6

MULTIPLY AND DIVIDE

1. Jovani made a long-distance call from Antigonish, N.S. to Moncton, N.B., on Sunday at 9 a.m. He talked for 7 minutes. What did the call cost him if the rate was $0.58 per minute?

The cost of the calls were $4.06.

2. Martha wants to make $55 this week. Her part time job pays $2.75 per hour. How many hours must she work?

$$55\overline{)275}^{\;5}$$

She has to work 5 hours.

3. While hauling a truck load of oranges, the transport truck was involved in an accident and spilled 2604 oranges. How many dozen oranges were spilled?

$$\begin{array}{r} 217 \\ 12\overline{)2604} \\ 24 \\ \hline 20 \\ 12 \\ \hline 84 \\ 84 \end{array}$$

2.17 dozen oranges were spilled.

4. For a championship football game between the Wildcats and the Reddogs the admission charge was $2.25. The price included the bus ride to and from the stadium. How much did it cost 347 people to attend?

$$\begin{array}{r} 3.47 \\ 2.25 \\ \hline 1.32 \end{array}$$

$1.32 That's how much it cost to attend.

5. A football stadium has seating for 68300 people. If 84% of the seats are occupied, how many people are at the football game?

$$\begin{array}{r} 800 \\ 84\overline{)68300} \end{array}$$

800 people are at the football game.

Figure 6:62 cont'd

6. A class of 18 Math 15 students is planning a field trip to New York. The total cost of the trip is $5760.00. If the school gives $900 towards the cost of the trip, how much will each student have to pay to go on the trip?

$$\begin{array}{r} 61.00 \\ 900\overline{)5760.00} \\ \underline{5600} \\ 1600 \\ \underline{900} \\ 700 \end{array}$$

$61.00 will each student have to pay to go on the trip.

7. An apple has 28.4 mg of vitamin C. A banana has 11.6 mg of vitamin C.

a) How much more vitamin C is there in an apple than in a banana ?

$$\begin{array}{r} 28.4 \\ \underline{11.6} \\ 6.8 \end{array}$$

6.8 VitaminC there is in an apple than in a banana.

b) How much vitamin C would you get if you ate 2 bananas and half an apple ?

37.2 Vitamin C. That's what you get.

point. The correct operation was used in question 7 but there was an error in her procedure.

A wrong operation was used in questions 4, 5 and 6. Subtraction was used instead of multiplication in question 4. Once an answer was reached there was no verification as to whether the answer made sense. In question 5, division was used instead of multiplication. The percentage problems proved to be very difficult. Ashley did not know what "84% of" meant. She knew that there would be less than 68 300 people at the football game, so she divided. Question 6 was a double operation question in which the first operation was subtraction. Ashley performed division.

Ashley was unable to find the pattern in "Complete the Pyramid". She saw that the left and right side of the pyramid were identical and filled those two columns in correctly. The rest of the pyramid was filled in with Carmen's help. Carmen drew triangles on Ashley's paper to show her how to solve the problem. Once Ashley knew what to do, she was able to finish the diagram.

Interpretation of Ashley's Work

Ashley was a mild-mannered young lady who liked to socialize with her friends. She was not very interested in school or mathematics. She liked to be in the teaching experiment because the group was small and the atmosphere was different from a normal classroom.

Ashley's procedural skills were not very well developed. She had difficulty with most of the worksheets but she didn't become impatient, frustrated or upset. She completed the worksheets the best that she could with the procedures that she had. If there was something that she felt she could not do, she did not complete that work or she asked her groupmates. She did not check with the teacher/researcher if her answer was correct; the answer she had was good enough. It was not important whether it was the correct answer.

Ashley had difficulty with all the worksheets. Her shining moment was when she found a way to compare decimal numbers by using money as her referent.

Ashley could be placed at level II of the procedural layer. She had just enough of a procedural knowledge base to be somewhat successful when dealing with decimal numbers. However, her lack of motivation, interest and emotions about school and mathematics will make it difficult for her to peel the procedural layer. Her procedures could be reinforced but she will not attain the abstract layer.

Ashley's post-test score (28%) and her delayed post-test score (33%) were below the post-test and delayed post-test means. This indicated her lack of correct procedural knowledge although her score increased slightly on the delayed post-test.

Interpretation of Group's Work

The work of the eight students who took part in the teaching experiment has been interpreted in light of the Onion Model. This interpretation was based on analysis of the interview data, the worksheet responses, the interaction during the teaching experiment and the field notes taken by the teacher/researcher. These data indicated that the students possessed a hodgepodge of procedural knowledge and that for certain students this grew to a more correct and more organized procedural knowledge structure than for others.

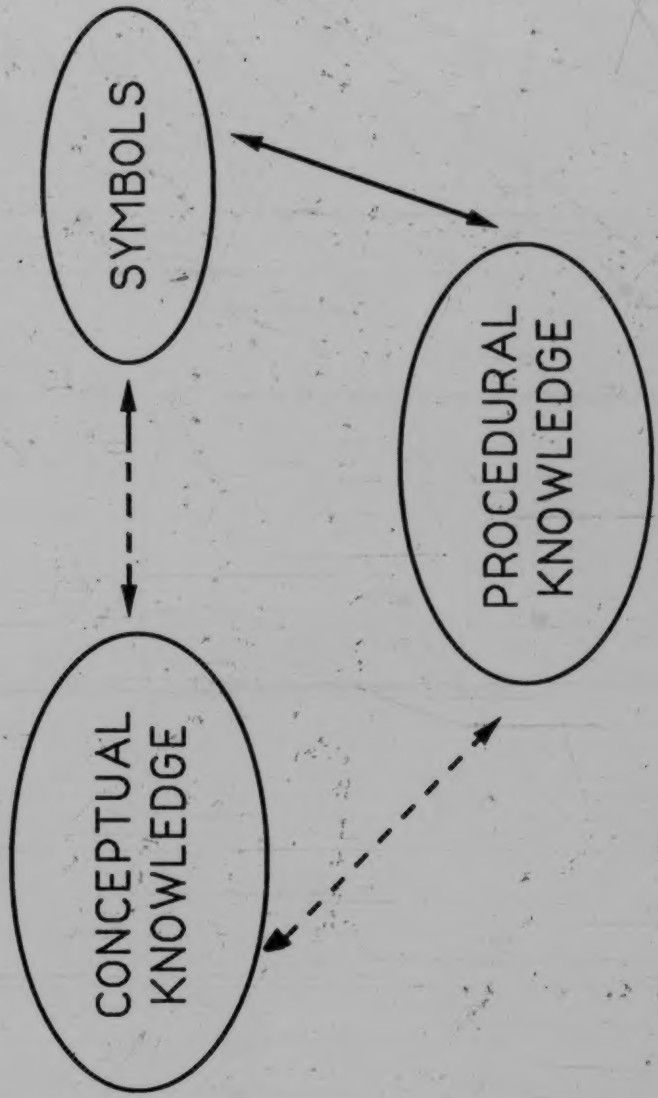
The students constant use of the word do ("I can't do this." "How do you do this?" "Can you show me how to do this?") demonstrated that their main concern was finding an answer and knowing which procedure to use and how to use it correctly. This concern for doing appeared to inhibit thinking about the situation in any way except in symbolic form. This often led to the application of disorganized symbolic procedures to a situation in which they did not apply.

Such incorrect and disorganized procedures suggested that these students may not have peeled the first two layers, as suggested in the model, before arriving at the procedural layer. The students demonstrated that they had a procedural base of decimal number knowledge but that there was nothing attached to this base. Therefore, when they forgot a procedure they had no where else to go to retrieve it or reconstruct it. It is as if a probe had been inserted at the procedural layer and bypassed the first two layers.

The procedural knowledge of these students was always connected to the symbols and procedures were not necessarily connected to conceptual knowledge. Figure 6:63 shows the relationship between the symbols, procedural knowledge and conceptual knowledge. The students possessed a bank of procedural knowledge (correct or incorrect) which was strongly linked to the symbols and vice versa. The symbols are a necessary part of procedural knowledge because it is impossible to perform procedures without them. They are the link between the decimal fractions and the procedures. However, in order for there to be a strong link between procedural and conceptual knowledge, there must be a strong link in two places: between the symbols and procedural knowledge, and between the symbols and conceptual knowledge. These students did not have a strong link between the symbols and conceptual knowledge and therefore did not have a strong link between procedural knowledge and conceptual knowledge.

The decimal number knowledge of these students does not seem to have grown beyond the procedural layer yet they possessed different levels of procedures. The onion metaphor would suggest that there are layers within layers with which various student work can be associated. Some students showed insights concerning decimal numbers that would perhaps permit them to go beyond the procedural layer. These

Figure 6:63
The Relationship Between Conceptual Knowledge, The Symbols and Procedural Knowledge


SYMBOLS
PROCEDURAL KNOWLEDGE
CONCEPTUAL KNOWLEDGE

insights allowed the work of these students to be placed closer to the abstract layer whereas other students who did not display insights into the decimal number system were placed at a lower level of the procedural layer.

The social and emotional characteristics of these Math 15 students play a role in determining whether the students will be able to peel the procedural layer or not. These students have strong emotions and feelings of frustration, anxiety and failure when they are in a school setting. The majority of these students have already experienced failure during their years of schooling and this emotion is still part of their school life. The feeling of being a failure seems evident when they constantly want to know what to do and how to do it. For these students procedural knowledge is knowing and may represent a form of conceptual knowledge to them. Therefore for some of these students it may be possible to peel or go beyond the procedural layer but it may be more difficult for others.

Chapter VII

Summary and Conclusions

Summary and Discussions

In this study efforts were made to understand the kinds of techniques that high school students with a long history of poor performance in mathematics, here called Math 15 students, used when involved in solving decimal computation problems; and the growth of decimal knowledge of these students under a particular teaching regime.

The purposes of the study were:

1) to ascertain the decimal number knowledge of students enrolled in Math 15, and from this base and under particular conditions,

2) to study students' knowledge under instruction during the teaching experiment,

3) to identify inter-relationships between held knowledge of students and individual knowledge growth under instruction, and

4) to try to characterize such knowledge and such inter-relationships theoretically.

Characteristics and Activities of the Study

To meet the purposes above the research activities occurred in three phases.

1) To ascertain the decimal knowledge held by Math 15 students and their views of such knowledge, Math 15 students were engaged in a decimal knowledge pretest and pretest interviews.

2) Eight volunteer students participated in a teaching experiment in which their activities in relearning decimal knowledge were closely monitored.

3) To ascertain decimal knowledge held by these students after the teaching experiment, a post-test and delayed post-test were administered. To try to relate their views of decimal knowledge to their thinking after the teaching experiment, delayed post-test interviews were conducted.

The purpose of the study was to look at how students' knowledge of decimals changed in the context of instruction. To do this it was necessary to ascertain the students' decimal number knowledge before the teaching experiment. This was done by means of the pretest and pretest interviews. The pretest served to identify a base level of understanding and application of decimal concepts tested. The written pretest (Appendix A) included items on decimal computation and a variety of items on the different meanings that can be attributed to decimal numbers such as: the relationship to fractions, place value and measurement. The pretest interviews (Appendix B) provided information on the students' perception of the pretest questions, and the procedures and processes used to solve the questions. Twenty-nine students took the pretest and nine students participated in the pretest interviews. Both were done under the conditions of anonymity of the students and were used simply to provide explanations of these Math 15 students' knowledge base of decimals.

Eight students, of the nine student interviewed, volunteered to be part of the teaching experiment which was conducted by the teacher/researcher and consisted of teacher developed and assigned activities. Instruction during the teaching experiment did not involve any direct teaching by the researcher but was characterized by students working together on the worksheets in small groups. These activities were designed with the pretest and interview information in mind. The intent of the worksheets was to provide meaningful contexts for students to encounter opportunities to address weaknesses in their knowledge of place value, scale reading, operations, estimation and fraction-to-decimal translation. Based on daily interactions and on field notes, on-going instruction was designed to meet particular student needs.

The purpose of the small groups was to allow the students to discuss their responses with each other and comment on each other's work. This was done in an attempt to involve the students in reflecting on their own misconceptions and errors, and to encourage them to reflect, debate and resolve any inconsistencies revealed by different answers. During this time the teacher/researcher interacted with the students on the mathematics and made field notes of what was going on within the groups, and of individual student's work on the mathematics. These notes were transcribed and studied daily in order to identify approaches and changes in approaches used by the students as well as to identify useful researcher questions for the next class. These interactions allowed the teacher/researcher to probe into the students' thinking processes and to propose other strategies that would help the student arrive at revised knowledge of decimal fractions. Notes from these interactions formed part of the basis for analysis of students' knowledge and its growth.

The openness of the teaching experiment which continued for fifteen sixty minute classes pleased the students and allowed them to mingle socially with each other while working together. The social aspect of the teaching experiment was important to these students because they felt relaxed and not pressured by the teaching experiment or the teacher/researcher.

After the teaching experiment a post-test (Appendix C) was administered. The post-test was used to check whether students learned particular techniques and decimal concepts during the teaching experiment. After a further ten weeks, during which time the students did not receive any additional instruction on decimals, a delayed post-test (Appendix D) was administered. The delayed post-test was comprised of the questions that were identical on the pretest and the post-test. The delayed post-test once again tested particular decimal knowledge and was used to assess the continuing status of decimal knowledge for students participating in the teaching experiment. One month after the delayed post-test, the students were interviewed. These interviews allowed the teacher/researcher to compare the conceptual knowledge held by students long after the teaching experiment to the conceptual knowledge of the students as it developed during the teaching experiment.

Although pretest, post-test and delayed post-test results were compared to assure that the instruction during the teaching experiment had some positive effect on achievement, the major analyses came by reviewing the interviews and the field notes collected about each of the students.

Results of the Study

The pretest and the pretest interviews were used to ascertain the decimal number knowledge of Math 15 students. The pretest contained questions on six categories: 1) place value, 2) estimation, 3) operations, 4) scale reading, 5) comparing decimals, and 6) fraction-to-decimal translation.

The analysis of the pretest and the pretest interviews indicated that these students possessed a hodgepodge of decimal knowledge with a strong procedural tendency. For example, the operations of multiplication and division were thought of as the "counting and moving of decimal places". Reading scales subdivided into tenths was far easier than reading scales subdivided into fifths or hundredths because the students relied on counting the number of marks or spaces between numbers. Syntactic translations were used for fraction-to-decimal translations; for example, the fraction 1/2 was written as 1.2. Comparing decimal numbers became a matter of counting the number of digits after the decimal point.

Having identified difficulties such as those above, a teaching experiment was planned. The aim of the teaching experiment was twofold: 1) to gain further evidence concerning the procedures and processes used when working with decimal numbers, and 2) to study students' knowledge as it changed under instruction.

Did the students change or grow during the teaching experiment? Before looking at the quality and characteristics of such growth, let us consider performance on post-test items to examine what students could do after instruction.

The post-test and the delayed post-test contained items on the following six categories: 1) scale reading, 2) operations, 3) comparing decimals, 4) problem solving, 5) estimation and 6) fraction-to-decimal translation. The delayed post-test was comprised of 50 questions that appeared on the post-test. Table 7:1 showed the results per category for each student on the post-test and delayed post-test.

At the end of the teaching experiment the students seemed more comfortable with the categories of operations and fraction-to-decimal translation. The pretest interviews indicated that the students could perform well on the addition and subtraction of decimal numbers but had difficulty with the multiplication and division of decimal numbers. The post-test and the delayed post-test results indicated that the students had been able to organize their rules for multiplication and division and could now perform those two operations much better. The same can be said for fraction-to-decimal translation. The teaching experiment allowed them to examine their misconceptions of transforming a fraction to a decimal syntactically. The delayed post-test interviews indicated that the majority of the students had been able to relearn the rule of dividing the denominator into the numerator. Thus, although the students still were rule oriented, their rules allowed them to think of decimal numbers as quantities which arose out of division.

The performance on scale reading remained approximately the same from the post-test to the delayed post-test. While performance on scale reading did increase slightly from the pretest to the post-test, the students still exhibited difficulty with the reading of scales divided into units other than tenths.

Performance also increased on the comparing of decimal numbers. That is, while before the study a student might have miscompared 1.4 and 1.25 by counting digits, during and after the teaching experiment they used place value ideas. This change may have been due to Ashley's newly discovered strategy. During the teaching experiment Ashley discovered that if she used money to represent decimal numbers she got the correct answer. She shared her strategy with the other members of the study.

The performance in the categories of estimation and problem solving remained poor throughout the study. In both the post-test and the delayed post-test the students were counting decimal places to estimate the answer to a multiplication item. The rule of "counting decimal places when multiplying" remained a strong part of their conception of decimals. Owens and Haggerty (1987) conducted a teaching experiment with much younger students and also found that students rely heavily on rules with one of those rules being the counting of decimal places when multiplying decimals. The post-test and the delayed post-test also indicated difficulty with problem solving. The delayed post-test interviews confirmed the difficulty students were experiencing when confronted with a written problem requiring the interpretation of a problem, and the choice of an operation to arrive at a correct operation. The student would try to get hints from the written problem. They would look for words like, "less" and "more" but still would not try to make quantitative sense of the problem.

Table 7:1

		Question Type					
		A	B	C	D	E	F
Jennifer	Post-Test	1/14	10/12	3/11	0/4	1/3	1/5
	Delayed	0/14	12/12	7/9	0/1	3/3	4/5
Ashley	Post-Test	2/14	9/12	2/11	1/4	0/3	0/5
	Delayed	4/14	9/12	2/9	0/1	0/3	0/5
Paul	Post-Test	13/14	10/12	7/11	0/4	0/3	5/5
	Delayed	14/14	11/12	7/9	0/1	0/3	5/5
Jessica	Post-Test	4/14	4/12	6/11	1/4	0/3	4/5
	Delayed	7/14	10/12	3/9	0/1	0/3	4/5
Julie	Post-Test	12/14	12/12	11/11	0/4	0/3	5/5
	Delayed	11/14	12/12	6/9	0/1	0/3	5/5
Carmen	Post-Test	3/14	8/12	3/11	1/4	0/3	0/5
	Delayed	2/14	8/12	0/9	0/1	0/3	2/5
Tyler	Post-Test	8/14	9/12	5/11	2/4	0/3	3/5
	Delayed	--	--	--	--	--	--
Ryan	Post-Test	7/14	10/12	2/11	0/4	0/3	0/5
	Delayed	--	--	--	--	--	--

A (Scale Reading)
B (Operations)
C (Place Value/Comparing Decimals)
D (Problem Solving)
E (Estimation
F (Fraction-to-Decimal Translation)

In summary, if one looks only at performance change over the course of the study, there was growth. This growth can be examined across the pretest, post-test and delayed post-test (Appendix K). This increase in scores indicates that the teaching experiment did make a difference. The teaching experiment corrected some of the incorrect procedures yet some of the incorrect procedures and decimal misconceptions were so deeply embedded into the students' knowledge of decimals that they were not corrected. The notion of counting decimal places when multiplying decimal numbers was such an example.

The interviews which took place one month after the delayed post-test suggested that the increased scores were due to a more organized and complete procedural knowledge. Their hodgepodge of decimal knowledge had been rearranged so that some of their procedures had been corrected. During the delayed interviews the students displayed more correct procedures than during the pretest interviews but they were not able to think about problems and problem solutions in any other way except procedurally. Still, this new capability enabled the students to feel more comfortable when performing decimal computations.

Even though the students displayed more correct procedures on the delayed post-test, their conceptual knowledge remained poor. For example, most students still insisted: 1) on relying on the counting of decimal places when multiplying decimals, 2) that multiplication makes bigger and division makes smaller, and 3) that reading a number on a number line was counting spaces. Although many students gained a natural place value strategy for comparing decimals, students still displayed difficulties when comparing decimals.

The new knowledge organization shown by the students can be related to the experiences in the teaching experiment. The personal work on activities did allow them to rethink their computations and comparisons of decimals. But the quality of inter-student interactions which was based on "getting the answer" likely hindered more conceptual development.

Reflections on the Teaching Experiment

One of the goals of the study was to examine students' knowledge of decimal numbers as it changed during a teaching experiment. The eight students who took part in the study were closely monitored over the duration of the teaching experiment. The teacher/researcher prepared worksheets based on the data collected from the pretest and the pretest interviews and hoped that the student would work on the worksheets in groups and reflect and discuss errors and misconceptions as they arose.

The teacher/researcher attempted to understand the insights provided by the students that would explain the various misconceptions as well as changes in knowledge.

The teaching experiment was designed to have an effect on the decimal number knowledge of the students. The worksheets and the interviews focused on the students' and not the teacher/researcher's interpretation of the task. In this sense the study was student oriented as the teacher/researcher took her cues from the students.

There were slight discrepancies between the teaching experiment as planned and the teaching experiment as happened. These discrepancies (discussed in Chapter III) did not invalidate the goal of the study. The students enjoyed being in small groups as this allowed them to mingle socially which is very important to these students. It was hoped that the students would work as a group on the worksheets rather than individually and that the discussions within the groups would provide insights into the thinking process held by the students. The insights gained from the student discussions were not as in depth as hoped, even with the interaction of the teacher/researcher. However insights from discussions did give further evidence as to the misconceptions held and how they were being used by various students. For example, even under instruction, which allowed the students to reorganize some of their distorted procedures, difficulties still existed with the multiplication rule being used for discussion; the counting of decimal places in multiplication; and reading a number line subdivided in units other than tenths.

The researcher feels that the teacher directed worksheet approach was a good one. The students completed the worksheets, discussed the various responses within the group in order to come to a consensus concerning the correct answer. However, very little conceptual reconstruction seems to have occurred. The students in the study seemed to be behaving at a procedural level with very little conceptual understanding of the decimal number system and very little interest in going beyond the correct procedure. Perhaps the social characteristics of these students did not allow them to be interested in or to go beyond procedural understanding.

The results of the pretest and the pretest interviews of the present study show that the students who participated in the present study, although at the high school level, are experiencing difficulty with the syntax and that age has not made them that much more proficient in their understanding of the decimal number system or in their use of rules and procedures. The results of this present study indicate that these students can be re-taught correct procedures and that this alone gives them a sense of understanding as they know understanding.

In summary, we have seen that these students came to the teaching experiment with a hodgepodge of syntactical rules. The teaching experiment allowed these students to reorganize their rules into a workable order which seemed to continue to be accessible long after instruction.

Situating the Findings

The remainder of this chapter is devoted to relating the findings noted above to other research on decimal knowing in children and to further analysis of these findings through the lens of the Onion Model.

The results of other research (Carpenter, 1981; Brown, 1981; Hart, 1981) done on the decimal knowledge of students, show that students perform poorly on tasks that require decimal computation and that most often the students will try to retrieve a poorly remembered computational rule that can be applied to a particular problem. The work of Hiebert and Wearne has been the most comprehensive in the study of procedural and conceptual understanding of decimals. The term "understanding" has itself been discussed extensively. For example, Hiebert and Wearne (1986) speak of procedural and conceptual knowledge while Skemp (1976) speaks of instrumental and relational understanding. Whether it be procedural, conceptual, instrumental or relational understanding, researchers agree that to have a complete understanding of decimals one must have a conceptual understanding of decimals. Hiebert and Wearne (1985) hypothesized that many students compute by relying solely on syntax-based rules and that semantic knowledge has no effect on performance. The pretest interviews, the attitudes and work of students during the teaching experiment and the delayed post-test interviews in this study confirm Hiebert and Wearne's research findings. The Math 15 students participating in this study were so rule-governed that they were not interested in knowing why a procedure works. They were mainly interested in knowing what rule to apply when and how to apply it. The insight provided by the results of this study is that students' procedures (even in this high school group) are indeed syntactically based. However, post-test results indicated that these students, even with their past history of poor mathematical performance, did grow to the extent that they knew when and how to perform most decimal computations correctly. The delayed post-test interviews also indicated that the increase in these scores, at least under this particular teaching regime, was due to an increase in procedural facility rather than from a better conceptual understanding of the procedures.

Hiebert and Lefevre (1986) suggest that even in junior high school and beyond, conceptual knowledge and procedural knowledge of students continue to develop along separate tracks but the focus remains on procedural knowledge. It was a formidable task for the students in this study to remember what rule to use when and use it correctly. As a result, at the outset they possessed a hodgepodge of disorganized, partially correct procedures. When trying to recall a rule or procedure to deal with a certain situation they would confuse more than one procedure or apply the procedure incorrectly. For example, the division of decimals was dealt with in the same way as the multiplication of decimals because they could not remember the rule for division of decimals but could remember the rule for multiplication of decimals. They therefore applied a rule they knew, which was incorrect in the given situation. This indicates, that for these students in high school, conceptual knowledge and procedural knowledge were indeed developing along separate tracks as suggested by Hiebert and Lefevre (1986) and that the focus still remained on procedural knowledge. For the students in this study, this focus on procedural knowledge carried over into the teaching experiment and even after the delayed post-test.

The results of the present study are also similar to those of a teaching experiment conducted by Owens and Haggerty (1987). Their results indicated that children can and do learn rules for computation. Owens and Haggerty raise the question about the quality of understanding that can be expected for children in grade 5 under normal classroom conditions and suggest that grade 5 students may not be mature enough to have a complete understanding of decimals. The results of this present study suggest that perhaps we should question the quality of understanding that exists in Math 15 students. These particular students may be unready for a complete understanding of decimals in a different sense. To begin with, these students have a mathematical background which is made up of a hodgepodge of partially correct procedural rules and their social characteristics are such that they have very little patience to even try to go beyond a procedural understanding of decimals.

Reflections on the Onion Model

We can now look at the teaching experiment from the point of view of the model. This will be done for two purposes: 1) to use the model to reflect on the teaching experiment, and 2) to use the experimental results to reflect on the model.

The results of this present study demonstrated that procedures can be reinforced, but the question becomes, "Can students who have reached high school, with a history of poor performance in mathematics, and with a strong procedural tendency acquire conceptual knowledge?" Hiebert and Wearne (1986) suggest two ways in which conceptual knowledge can be attained: 1) by the construction of relations between pieces of information, or 2) through the creation of relationships between existing knowledge and new information that is being presented. The proposed Onion Model showed one way in which students might build such mathematical knowledge or make such connections. In what sense could the layers of the model be related to the background knowledge of the students and to their growth during the study?

By observing various behavior patterns that emerged, it was possible to reflect on the model. The proposed Onion Model suggested that conceptual knowledge cannot be attained without first having peeled all the layers within the model in a metaphorical sense. The layers represent different kinds of understanding acts. "Peeling a layer" means that a person has accomplished the kind of thinking necessary to perform the acts relevant to that particular layer. Peeling one layer at a time would allow one to reach the core as would peeling more than one layer at a time. Peeling more than one layer at a time would suggest that a person can use the kinds of thinking acts from more than one layer together. The core metaphorically represents having arrived at complete understanding after having peeled the kinds of understanding necessary to the previous layers of the model. There are other ways in which the metaphor may be interpreted. It may be beneficial for some students to peel one layer at a time yet for other students it may be beneficial for two layers to be studied together. For example, one student may do best by building concrete models first and building symbolic models. For another student, such understanding acts could best be developed together. Perhaps for yet other students slicing the model and touching all layers at the same time may be what is required for those students to arrive at conceptual understanding. Such multi-levelled activities might arise in a sophisticated problem solving environment which would allow students to use and analyze many decimal representation at once.

The results of this study suggest yet another interpretation and use of the model. The results indicated that while all students in this study were at the procedural layer, it appeared that they had not completely peeled the first two layers. They were able to use concrete referents for some situations but not for others. For example, they were able to order decimal numbers when they compared the decimal numbers to money. It is as if the first two layers were touched but not peeled completely and then a probe had been inserted at the procedural layer. If this is so, the question we can now ask is whether we can build onto this procedural knowledge and move the probe either farther into the model towards the core or towards the outside of the model to organize necessary concrete experiences. In other words, can we take developed procedural knowledge and build meaning beyond it?

Although the students have all been placed at the procedural layer (see Chapter VI), there were differences in the students' ways of thinking about the procedures, their meaning and their use. This data provoked the researcher to take another look at the model. Using the onion metaphor, one can see layers within layers. Therefore, as seen in Chapter IV, students could be placed on four different levels within the procedural layer. For example, although the two students below, both had organized procedural knowledge, they displayed differences which allowed them to be placed at different levels within the procedural layer. For example, Paul had a correct, complete and organized procedural knowledge structure which allowed him to feel quite confident when dealing with situations involving decimal numbers. Paul, during the delayed interview, demonstrated that he was interested in going beyond the procedures when he said, "When you multiply two decimal numbers the answer is smaller ... but when you divide the answer is bigger. I don't know why that is ... but it just is." Paul was placed at level IV of the procedural layer, close to the mental model layer. Paul displayed abstract insights concerning the properties of decimals as he saw them. Julie could do decimal computations, was confident of her procedures and could check the validity of her work. But unlike Paul she did not indicate that she saw any patterns in the computations or their results and was placed at level III of the procedural layer. The difference between level III and level IV was not in consistent performance or even in the way students saw decimals. Students at level IV such as Paul have intuitions about procedures and question the procedure; students at level III do not display these intuitions.

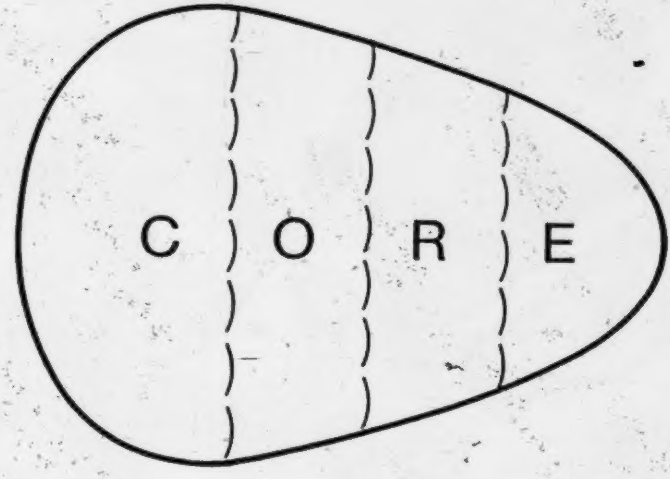
After the completion of the study and upon a certain amount of reflection, another question arose, "Is conceptual knowledge an all or nothing thing?" This study has portrayed it as such, yet perhaps it is not. Perhaps the core is not symmetrical as shown in Figure 2:1 but asymmetrical (Figure 7:1). If one thinks of the core as conceptual knowledge, perhaps parts of conceptual knowledge are arrived at in each of the layers as shown in Figure 7:1. Conceptual knowledge of decimals can occur concretely, procedurally or abstractly, although one would not say it was fully achieved until all the different thinking processes within all the layers had been attained. thus, perhaps some knowledge which looks procedural to the observer is conceptual to the student. Perhaps one can build onto this small piece of conceptual knowledge and finally link this to the more complex conceptual knowledge shown by the larger bulb on the end (Figure 7:1) Reflecting on this idea of conceptual knowledge it is possible to say that the students in this study gained in their conceptualization of decimals. Seven of the eight students in the teaching experiment appeared to have at least organized their knowledge of decimal procedures. These students progressed from disorganized, incomplete and incorrect procedures to a more organized, more complete and more correct procedural knowledge. Such better organized knowledge structure represents one form of understanding. That such understanding could lead to conceptual intuitions about decimal computations was also observed.

Concluding Statement and Reflections on Further Research

The students in this study were in a class, Math 15, of students with a long history of poor performance in mathematics. The initial ascertaining experiment consisting of a pretest and interviews with nine students, indicated that the students possessed a hodgepodge of syntactically oriented rules concerning knowledge of and operations on decimal fractions, the mathematics topics selected for this study. The interviews revealed that the students were simply interested in doing something to get an answer and frequently used inappropriate rules.

The results of the study indicate that the "how" is very strong in these students and persisted during instruction. The improvement in scores from the pretest to the post-test is due to a reorganization of previously learned procedures which would lead to a more correct procedural understanding. However, it is felt by the teacher/researcher that for some of these students at this level of schooling it would be possible to attain a more conceptual understanding. For these students, perhaps the reorganization or reinforcement of procedures exhibited in this study was a step towards the reconstruction of decimal knowledge. These students exhibited a desire to succeed in school and to reflect on their thinking. For them, the use of more concrete or more practical situations might allow them to connect their procedural thinking with broader concepts.

Figure 7:1
An Interpretation of the Core



It has also been observed that for other students reconstruction is difficult and conceptual understanding may be difficult to achieve. For these students, growth in understanding represented only a slightly more organized set of syntactic rules. These students seemed actually disinterested in anything but numerical performance using rules.

There are several explanations for the nature of change in understanding observed in the study and the difficulty in getting some students to go beyond the simple doing of procedures. Skemp (1989) would explain this by suggesting that these students had likely learned decimal knowledge in a rote procedural way. He would see these results as supporting his contentions that the practice of having students learn procedural knowledge first with hope of this knowledge growing into conceptual understanding later is faulty. Thus all of these students' instructional histories prompted them to treat instruction in the teaching experiment procedurally and inhibited any focus on why things worked. Of course, the results and interpretations show that these students are not much different from their contemporaries in matriculation mathematics or indeed beginning calculus at the university level who interpret doing mathematics as a procedural activity.

For some of the students in this study, their mathematical history was compounded by their approach to school. They have not been good at "school" and social and personal concerns inhibited them from participating in any instructional activities except in a minimal way.

Results of this study suggest that students in Mathematics 15 came to the restudy of decimal fractions with a hodgepodge of disorganized rules. They came with a history of rule oriented learning and with special concerns which act against their fruitful participation in instruction. Under the particular instruction in the teaching experiment, the students appeared to reorganize their procedural knowledge and appeared better able to see when to use a procedural rule as well as how. They were able to do decimal fractions task where procedures could be used. This "understanding" or better organized procedural thinking seemed detached from other kinds of decimal number knowing. Although the metaphor used in building activities and analyzing the results in this study suggested growth as a progression through layers, the information from the teaching experiment suggest that non-linear analyses may be more appropriate.

This study indicated that a certain kind of growth, in the students' understanding of when and how decimal procedures are used, can and does occur. Further research is needed to examine the question of the nature and existence of instruction which both motivates and provides the opportunity for such students to understand "why" decimal procedures work. Such research could also help clarify the general nature of growth of mathematical understanding, at least of decimal fractions, by various students with a long term history of poor mathematical performance. In such research the following considerations seem useful:

i) The individual group activity materials used in this study provided a learning environment well accepted by the students. However, future materials should contain questions which students see as open ended.

ii) The materials used in the study saw students using procedures in a variety of meaningful but non-concrete task settings. Future research should allow the students to use concrete representations (eg. multi based blocks, student constructed number lines) in both doing and explaining their decimal number thinking. Both the use and effects on the growth and nature of understanding could be studied.

iii) In this study calculators were not used. In future studies with such students, the use of calculators, both in relationship to students' procedural knowledge and their concrete activities, could be considered.

iv) In this study the teacher/researcher acted by assisting and discussing with students as they worked on the materials. In future studies, the teacher/researcher could take a more active teaching role.

Future research could also consider issues raised in this research such as:

i) Can, and under what circumstances can, students with a long history of poor procedural learning in mathematics, achieve conceptual understanding beyond the better organized and accessible procedural knowledge shown here?

ii) The growth in procedural knowledge shown here could be characterized using a layered model. Is growth in mathematical knowing, best characterized by a layered model such as the Onion Model? Would other interpretations be necessary to characterize growth in learning mathematical concepts for students such as these or students in general?

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Appendix A

Pretest

DECIMAL KNOWLEDGE MATH 15

CODE NAME: ______________________________

CALCULATORS MUST NOT BE USED

1. What does 9.7 mean?

Check (✓) the best answer from the following:

a) Ninety seven

b) Nine remainder seven

c) Nine hundred and seven

d) Nine and a seventh

e) Nine and seven tenths

f) None of these, I think it means ______________________________

____

2. a) CIRCLE the BIGGEST of these three numbers:

0.62 or 0.236 or 0.4

b) How can you tell it is the biggest ? ______________________________

____

3. Which of the following is thirty-seven thousandths?

a) 0.037

b) 0.37

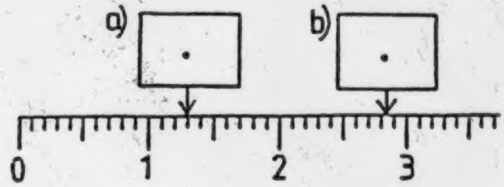
c) 37

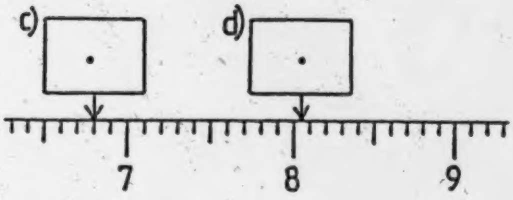
d) 37000

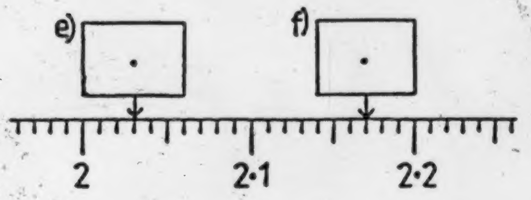
e) None of these. I think the answer is ____________________.

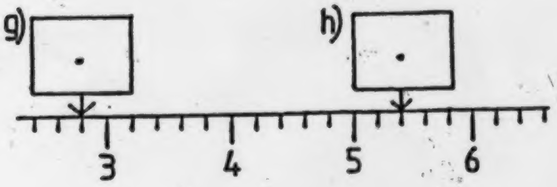
4. In the decimal number 0.248 the 2 stands for 2 ____________________.

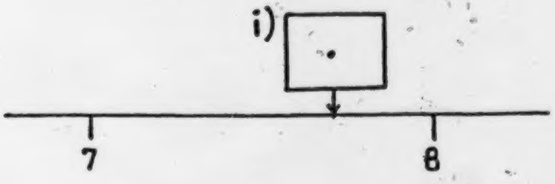
5. **Read the following scales and write your answers in the boxes provided. Give all your answers as DECIMALS.**



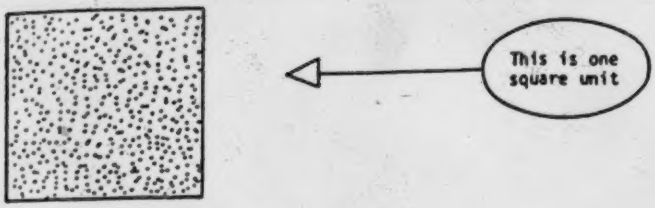






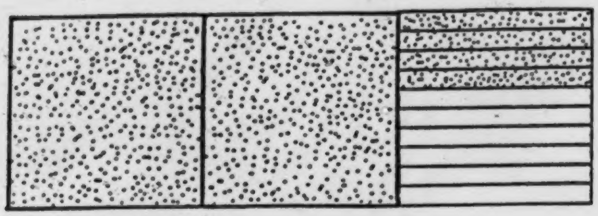


6.


This is one square unit

Give your answers as DECIMALS

a)

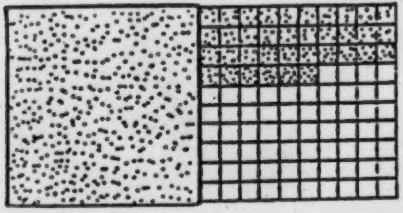


The area shaded

is .

square units

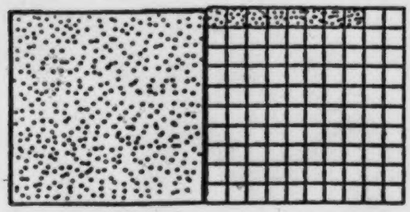
b)



The area shaded

is . square units

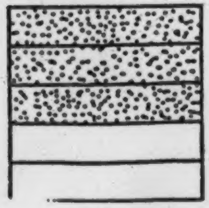
c)



The area shaded

is . square units

d)



The area shaded

is . square units

7. **Circle the one that gives the BIGGER answer.**

   a) 8 x 4 or 8 ÷ 4

   b) 8 x 0.4 or 8 ÷ 0.4

   c) 0.8 x 0.4 or 0.8 ÷ 0.4

---

8. **Four tenths is the same as ____________ hundredths.**

---

9. **Six tenths as a decimal is 0.6 . How would you write as a decimal three hundredths? ______________________________**

---

10. **Estimate the answer then place the decimal point in the given answer.**

    a) 2.42 x 3.610 = 08736200

    b) [illegible].20 ÷ .08 = 0040000

    c) .42 x .23 = 00096600

---

11. **Perform the following subtractions.**

    a) .78 - .35 = ____________________

    b) .60 - .36 = ____________________

    c) .86 - .3 = ____________________

---

12. **Perform the following multiplications.**

    a) 6 x .4 = ______________________

    b) 2 x 3.12 = ____________________

    c) 8 x .06 = ______________________

---

13. CIRCLE the BIGGEST number in each group of three :

a) 5436 or 547 or 56

b) 6.78 or 45.6 or 345

c) 3.521 or 3.6 or 3.75

d) 15.4 or 15.56 or 15.327

e) 4.09 or 4.7 or 4.008

---

14. CIRCLE the BIGGEST number in each pair.

a) 76 or 760 ....

b) 76 or 076 ....

c) 0.76 or 0.760 ....

d) 0.76 or 0.076 ....

e) 0.76 or .76 ....

Write SAME
if you think
they are the
same size

---

15. Write down any number which is

a) BIGGER than 3.9 but SMALLER than 4 ___________

b) BIGGER than 6 but SMALLER than 6.1 ___________

c) BIGGER than 8.9 but SMALLER than 8.15 ___________

d) BIGGER than 0.52 but SMALLER than 0.53 ___________

Write NO
if you think
there is no
answer

e) How many different numbers could you write down which lie between 0.52 and 0.53?

____

---

16. CIRCLE the number nearest in size to 0.16

0.1 0.2 15 0.21 10

CIRCLE the number nearest in size to 2.08

209 2.9 2.05 2.1 20.9

17. Perform the following additions.

a) 4.6 + 2.3 = ______________

b) 5.3 + 2.42 = ______________

c) 5.1 + .46 = ______________

---

18. Perform the following divisions.

a) 2.1 ÷ 3 = ______________

b) .24 ÷ .03 = ______________

c) .56 ÷ 7 = ______________

---

19. Circle the CALCULATION you would need to find the answer to the following problem.

My car can go 42 km on each litre of gas. How many kilometers can I expect to travel on 11.37 litres of gas?

a) 11.37 ÷ 42

b) 42 - 11.37

c) 11.37 - 42

d) 11.37 x 42

e) 42 + 11.37

f) 42 ÷ 11.37

---

20. Which decimal is equal to the fraction 1/5 ?

a) .15

b) .2

c) .5

d) .51

e) I don't know.

---

21. What are the decimal numbers for the following fractions?

a) 1/2 ______________

b) 2/3 ______________

c) 3/4 ______________

22. Write down the next 2 numbers in each sequence as decimals.

a) 0.2, 0.4, 0.6, ... , ... (adding on 0.2 each time)

b) 0.3, 0.6, 0.9, ... , ... (adding on 0.3 each time)

c) 0.92, 0.94, 0.96, 0.98, ... , ... (adding on 0.02 each time)

d) 1.13, 1.12, 1.11, ... , ... (taking away 0.01 each time)

---

23. a) Add 0.1 to 4.256 __________ d) Take away 0.1 from 15.835 __________

b) Add 0.1 to 3.9 __________ e) Take away 0.1 from 13 __________

Appendix B

Pretest Interviews

PRETEST INTERVIEW QUESTION

1. Which of these three numbers is the biggest:
0.62 or 0.236 or 0.4

How do you know it is the biggest?

2. Is there a number that is bigger than 4.9 but smaller than 5 ?

3. CIRCLE the number nearest in size to 0.16
0.1 0.2 15 0.21 10

4. Subtract:

i) .86 - .3
ii) 4.7 - .24
iii) 7 - .4

5. Multiply:

i) .34 x 2.1
ii) .05 x .4

6. Add:

i) 6 + .32
ii) 5.1 + .46

7. Divide:

i) .028 ÷ .4
ii) 3 ÷ .6

8. What is the decimal for:

i) 1/2
ii) 3/1000
iii) 1/5

9. Add 0.1 to 6.98

10. Take away 0.1 from 1.06

11. If we have the sum 5 + 2 = 7, we could say the following: John has 5 records. His father gave him 2 more for his birthday. Now, he has 7 records altogether.

What kind of a story can we have with the sum 4.6 + 5.3

= 9.9 ?

What kind of a story can we have with the multiplication 42.12 x .25 = 10.53 ?

Group 1
Nov.4, 1988

1. QUESTION: Which of these three numbers is the biggest: 0.62 or 0.236 or 0.4 ?

GROUP 1: Two students (one male, student #1 and one female, student #2)

1. St. # 1 (male): 0.62
St. # 2 (female): 0.4

Interviewer to St. #1 : Why do you say that 0.62 is the biggest?

St. # 1: Because the number in the tenths column is bigger.

Interviewer to St. #2: Why do you say that 0.4 is the biggest?

St. # 2: Because there's only tens. If you had hundreds and thoudands it would be less. (She used these terms rather than hundredths and thousansths.)

Interviewer: What is 0.4 ?

St. # 2: 4/10

Int: What is 0.6?

St. # 2: 6/10

Int: If 0.4 is the same as 4/10 and 0.6 is the same as 6/10, does that mean that 0.6 is greater than 0.4?

St. # 2: No, because you have a 2 after the six.

2. QUESTION: Is there a number that is bigger than 4.9 but smaller than 5?

Answer: Both students say no.

Int: Why not?

St. # 2: You could have 4.9 1/2.

St. # 1: That's not a decimal.

Int: What would 4.9 represent?

St. # 1: Almost all of 5.(pause) There´s nothing smaller. (Pause) I don´t know.

Int: Could 4.9 represent something that I could relate to?

(I asked the students what 4.90 reminds them of that they use in everyday life. Student # 2 immeidately said money. Then there was no problem to find a number < 5 but >4.9 .)

3. QUESTION: CIRCLE the number nearest in size to 0.16: 0.1, 0.2, 15, 0.21, 10

ANSWER: Both students said 0.21.

Int: Why is 0.21 the closest to 0.16?

St. # 2: It´s closest.

Int: How do you know it´s the closest?

St. # 1: It´s a shorter distance.

(Student # 2 agreed with student # 1.)

(Went back to money. Both students said 0.16 was like 16 cents. They both said that 0.1 was 1 cent. When a zero was added they saw that it was 10 cents and then saw that the correct answer was 0.2.

4. Int: How do I subtract decimals?
What would be the answer to .86-.3 ?

St. # 1: 0.56

(Student #2 was working it out on paper when St. #1 gave the answer quickly.)

Int: How did you find 0.56?

St. # 1: You subtract the tens.

(Student #2 didn´t comment.)

Int: What is the answer for 4.7-.24 ?

St. # 2: 4.46

St. # 1: 4.54

Int: How did you get 4.46 (to st. # 2)?
St. # 2: You have to put a zero in 4.7 (4.70) and subtract. (Student lined up the decimals and did the subtraction correctly).

(St. # 1 did:

$$\begin{array}{r} 4.7 \\ -\ .24 \\ \hline 4.54 \end{array}$$

He did not borrow.

Int: What would be the answer to 7-.4 ?
Both students said 6.6. (Both students had a correct procedure.)

5. How would I multiply .05 x .4 ? What would be the answer?

St. # 2: .020

St. # 1: 0.20

St. # 2 to st. # 1: That´s wrong, count how many decimal places.

St. # 1: Oh I knew that.

Int: Which is the correct answer?

St. 1 and 2: .020

6. QUESTION: How would I add 6 + .32?

St. 1 and 2: 6.32 (Both students had a correct procedure.)

QUESTION: How would I add 5.1 + .46?

St. 1 and 2: 5.56 (Both students had a correct procedure.)

7. QUESTION: What would be the answer to the division .028 ÷ .4?

(Student # 1 is having difficulty doing this on his paper. Student #2 has .07 on her paper.)

Int: How did you find .07 (to St. # 2)?

St. # 2: .4 divided into .028. You have to move the decimal in the 4 to the other side so that it´s just a normal 4. Then the decimal in front of the zero you move to the other side. So instead of .028 it´s .28 and then you divide.

Int: Where do I put my decimal in the answer?

St. # 2: On top of the decimal.

St. # 1: You count the number of decimal places? (He´s not sure about this and poses this as a question.)

St. # 2 showed St. # 1 where the decimal went and he said "Oh yeah, right." It´s hard to say whether he remembered the rule or not.

8. QUESTION: If I wanted to change the fraction 1/2 to a decimal, what would that decimal be?

St. # 2: 0.5

St. #1 did not make a response.

QUESTION: What about the fraction 3/1000? What would this be as a decimal?

St. # 1: .30

(Student # 2 is working it out on paper by division and she gets .003.)

Int: How did you do that?

St . # 2: 1000 divided by 3 (She then explained the procedure and did it correctly.)

QUESTION: What would be the decimal for the fraction 1/5?

St. # 2: .2

St. # 1: repeated .2 but he did not finish the division so maybe he was just repeating her answer because he had not finished the division.

9. QUESTION: What would be the answer for this question: add 0.1 to 6.98 ?

St. # 2: 7.08

Student # 1 agreed with St. # 2.

10: QUESTION: What would be the answer to this question: take away 0.1 from 1.06 ?

Both students said 0.96.

11. First Part of the question:

St. # 2 used a story with money where 4.6 = $4.60, 5.3 = $5.30 which together makes $9.90.

Int: Where else can I find decimals?

St. # 1: in prices

Int: Are there other places besidea money where I can find decimals?

ST. # 1: weight. Then used a story dealing with weight.

Second Part:

St. # 1 had a correct story.
42.12 kg at 25 cents /kilo meams that you pay 10.53.

St. #2 was not able to come up with a story.

Group 2
Nov.4, 1988

GROUP 2: Two students (one male and one female)

1. QUESTION: Which of these three numbers is the biggest: 0.62 or 0.236 or 0.4 ?

St. # 1 (male) & St. # 2 (female): the middle one (0.236).

Int: Why is it the middle one? How do you know?

St. # 1: There are more numbers on the end.

St. # 2: Thers´s more of a decimal to it.

(Both students agree that because 0.236 has 3 digits after the decimal [whereas the other decimal numbers have less digits] that it is the biggest. St. # 1 also mentioned that 0.4 only went to tenths, 0.62 only went up to hundredths but 0.236 went up to thousandths. Both students agreed that I could fill in empty spaces with a zero. Then they saw that o.62 was the biggest.)

2. QUESTION: Is there a number that is bigger than 4.9 but smaller than 5?

St. # 2: No

St. # 1: on the test I put 4.90

Int: Is 4.90 > 4.9 ?

St. # 1: yes

Int: Why is 4.90 greater than 4.9?

St. #1: Because it has two numbers after the decimal point.

Int: Outside of school where could I use 4.90?

St: # 1: (Instantly) $4.90.

Int: What can I do with the 5?

St. # 1: $5

Int: If we think of money, is there a number >4.90 and <5?

St. # 1: $4.91

(Both students could relate to money.)

3. QUESTION: CIRCLE the number nearest in size to 0.16: 0.1, 0.2, 15, 0.21, 10

St. # 1: 0.21

St. # 2: a hesitant 0.21

Int: How do you know that it's 0.21 ?

St. # 1: I just saw that there was a zero and a decimal there and nothing else is close to .16 except for 15 but it doesn't have a point.

(When the problem was related to money both students agreed that .16 = 16 cents. St. # 2 said .1 was 1 cent. After filling the empty space with a zero, St. # 1 said that it was $1 then quickly said 10 cents. St. # 1 then related each decimal to money and then came up with 0.2 being the closest to 0.16. St. # 2 agreed but she hesitated. It is difficult to say whether she saw or just went along with St. # 1.)

4. Int: How do I subtract 2 decimals? What would be the answer to .86-.3?

St. # 1: You just subract it out and count how many places are after the decimals and then put the decimal in.

St. # 1: .53

St. # 2: 5.3

Int: (to St. #1) How did you get .53 ?

St. # 1: (did this on his paper)

$$\begin{array}{r} .86 \\ .3 \\ \hline .53 \\ 3\ 2\ 1 \end{array}$$

(counted three decimal places)

St. # 2: (did this)

$$\begin{array}{r} .86 \\ .3 \\ \hline 5.3 \end{array}$$

"I just looked diredtly up and put the decimal." (She is saying that she lined up the decimals but she didn't really line them up.)

The problem was once again related to money: You go to the store with 86 cents and buy gum for 30 cents, how much money do you have left ?

St. # 2 : (instantly) 56 cents

St. # 1: 53 cents

Is the right answer 53 cents or 56 cents ?

Student # 1 now said 56 cents but neither one could explain how to subtract decimals.

Int: What would be the answer for 4.7-.24?

St. # 1: 4.46

St. # 2: .23

(Related the problem to money and St. # 2 got it correct answer once it was related to money.)

5. QUESTION: How would I multiply .34 x 2.1 ? What would be the answer?

St. # 1: 7.140

St. # 2: 71. 40

Int: The numbers are the same but the decimal is in a different place. How did you know to put the decimal after the 7 (to st. # 1).

St. # 1: because you count the numbers after the decimal 1,2 3,4.

$$\begin{array}{r} 2.10 \\ \times \quad .34 \\ \hline 840 \\ 630 \\ \hline 7.140. \\ 4\;3\;2\;1 \end{array}$$

Int: How did you decide to put the decimal after 71?

St. # 2: Count the spaces after the decimal then I used 2 places (She became confused and could not explain her method.).

6. QUESTION: How would I add 6 + .32 ?

St. # 1: The answer would be 38 cents

St. # 2: $9.20

St. # 1 was using 6 as 6 cents and .32 as 32 cents which made 38 cents.

St. # 2 (even though she asked if 6 was like $6 she was still unable to add correctly.)

Int: How do I do this on paper.

St. # 2 was able to do it correctly on paper.

(SOMETHING WENT WRONG WITH THE TAPE HERE.)

7. QUESTION: What would be the answer to this division: 3 ÷ .6 ?

St. # 1: .2

St. # 2: (student has on her paper $.6\overline{)\,3.}$)

Neither student knew the procedure for dividing decimals.

8. QUESTION: If I wanted to change the fraction 1/2 to a decimal, what would the decimal be ?

St. # 2: 0.5

St. # 1; 1.2

St. # 2: half is always .5

St. # 1: It can be both maybe (.5 and 1.2). (He did not have any idea of 1/2 as a decimal.)

Related it to half a dollar. Is $1.20 half a dollar? They didn't know this at all. Could not do it from a procedural point of view.

Int: If I didn't know that 1/2 was .5 how would I find it ?

St. # 2: divide

Int: Divide what?

St. # 2: 10 into 50

St. # 1 kept stating that 1/2 was 1.2 even after saying 1/2 a dollar was 50 cents which was .5 of a dollar. They did not know how to change a fraction to a decimal.

QUESTION: What would be the decimal for the fraction 1/5 ?

St. # 1: .2

St. # 2: So does that mean that we have to divide the bottom into the top ? (had no idea of how to proceed.)

11. First part of the question:

St. # 2: gas

St. # 1: yeah, liters, gas.

St. # 2: What about ratings in a bank? Interest?

St. # 1: money

Second part of the question:

St. # 2: kilometers

St. # 1: money. Mike gave John $42.12 and he found 25 cents and put it in his pocket. He pulled it out and had $10.53.

Int: He starts with $42 in his pocket and ends up with $10 ?

St. # 1: OH!

Int: (to st. # 2) You said km. What kind of a story can

I make with km?

St. # 2: My dad drove 42.12 km and then another 25 miles and altogether we drove 10.53. (She has mixed up km and miles in the same sentence).

Int: Let´s say I have $42.12. In what situation would I multiply by .25 ?

St. # 2: Family allowance

Int: How would that happen?

St. # 2: It´s $42

Int: Why would I multiply by .25 ?

St. # 2: How many cheques do you have?

Neither student had any idea of why you would multiply by .25. They have no idea of multiplication by a decimal or a fraction. They are of the opinion that multiplication makes bigger.

Group 3
Nov. 8,1988

GROUP 3: 2 females

1. QUESTION: Which of these three numbers is the biggest: 0.62 or 0.236 or 0.4 ?

St. # 1: 0.236

St. # 2: 0.236

Int: Why is 0.236 bigger than the other two numbers ?

St. # 1: Because after the decimal it´s the biggest. 0.4 has only one number after the decimal, 0.62 has 2 numbers after the decimal but 0.236 has three numbers after the decimal.

(Both girls agreed that it was the number of digits after the decimal that decided the size of the decimal.)

2. QUESTION: Is there a number that is bigger than 4.9 but smaller than 5 ?

St. # 1: 4.10

St. # 2: I don´t know.

Int: Are there any numbers between 4.9 and 5 ?

St. # 2: Yes

Int: What would come after 4.9 ?

St. # 1: 4.10 because 10 comes after 9.

(The problem was then related to money. One student said that 4.90 represented 490. The other student said that it could represent the price of something. 4.90 could represent $4.90 and 5 could represent $5.

Int: Are there any numbers between $4.90 and $5 ?

St. # 2: 91, 92

(Even with money they still had difficulty answering this question.)

3. QUESTION: CIRCLE the number nearest in size to 0.16:

0.1, 0.2, 15, 0.21, 10

St. # 1: Does it have to be a decimal number?

Each number was related to money. They were very confused. St. # 2 said that 15 represented 15 cents because there wasn´t a decimal.

St. # 1: 0.10

St. # 2: 0.21

St. # 1: They´re both the same.

St. # 2: No they´re not. 0.21 is the nearest.

Int: How do you know?

St. # 1: There´s a difference of 5 from 0.16 to 0.21 and a difference of 6 from 0.16 to 0.10.

Int: What about the difference between 0.16 and 0.20 ?

St. # 2: But 0.20 isn´t one of the numbers. (She couldn´t relate 0.2 and 0.20.)

St. # 1: I think it´s 0.2.

St. # 2: But wouldn´t 0.21 be the closest. (Because the zero was not in 0.2 to make 0.20, she could not see it as the answer because it only had one digit after the decimal point whereas 0.16 had 2 digits after the decimal.

4. QUESTION: How do I subtract decimals ?

St. # 1: You line up the decimals (Hesitated) No, you don´t have to. (she is trying to remember rules but she has them mixed up because she does this problem as a multiplication.)

Int: What would be the answer to .86-.3?

St. # 1: 0.83

St. # 2: 0.83

Int: How did you get 0.83 ? (to st. # 2)

St. # 2: I subtracted 86 take away three makes 83.

Count 2 decimal places and put the decimal.

$$\begin{array}{r} .86 \\ .3 \\ \hline .83 \end{array}$$

St. # 1: That's what I did.

$$\begin{array}{r} .86 \\ .3 \\ \hline .083 \\ 3\;2\;1 \end{array}$$

(St. # 1 now has an answer of .083 because she counted three decimal places.)

When the problem was related to going to the store and buying gum, St. # 1 said that the answer would be 56 cents.

Int:(to st. # 1) You said when you began this problem to line up the decimals. What did you mean by that?

St. # 1: No, that's for multiplying.

St. # 2: (agreed with st. # 1 that lining up the decimals is for multiplying and dividing.)

St. # 2 came to the conclusion that 0.83 was the correct answer because you count the number of decimal places. There are three decimal places so you move the decimal in the answer three places.

4. QUESTION: What would be the answer to 7-.4 ?

St. # 1: .3

St. # 2: .3

(St. # 1 is really concerned about moving the decimal a certain number of places.)

5. QUESTION: How would I multiply .05 x .4 ? What would be the answer?

St. # 1: 0.020

St. # 2: 0.20

Int: Are those the same number ?

St. # 1: Yes.

St. # 2: They are but the decimal is in a different place.

(St. # 1 did the multiplication correctly.)

St. # 2 said she moved the decimal three places and realized her answer should be .020.

(They said that moving decimal places was for adding and subtracting but they are using that rule here with the multiplication.)

6. QUESTION: How would I add 6 + .32 ?

St. # 1: .38

St. # 2: .38

When the problem was related to money st. # 1 said that the answer should be $6.32. They can't relate decimals to an everyday experience.

QUESTION: How would I add 5.1 + .46 ?

St. # 1: 5.56

St. # 2: 5.56

(They got the correct answer with a hint about lining up the decimals.)

7. QUESTION: What would be the answer to the division .028 ÷ .4 ?

St. # 1: 7

St. # 2: 7

Int: How did you find 7 ?

St. #1: Just divide 28 by 4.

Int: What happens to the decimal point?
St. #2: It just disappears.

8. QUESTION: If I wanted to change the fraction 1/2 to a decimal, what would that decimal be ?

St. # 2: 1.2

St. # 1: I don´t know. (Then she used division to find the answer.)

St. # 1 did this:

$$\begin{array}{l} \quad\ 5\ r\ 8 - 5.8 \\ 2\overline{)10} \\ \quad\ \underline{2} \\ \quad\ 8 \end{array}$$

St. # 1: I don´t use decimals. I just add zeros.

St. # 2: Does the 2 always go in front of the 1 ?

(They are both very confused about the knowledge and procedures of decimals.)

9. QUESTION: What would be the answer for this question: add 0.1 to 6.98 ?

St. # 1: 7.08

St. # 2: 7.08

(Both students had a correct procedure.)
10. QUESTION: What would the answer to this question be: take away 0.1 from 1.06 ?

St. # 1: 0.105

St. # 2: How did you get that?

St. # 1: I didn´t line them up. (She still couldn´t get the right answer and is very concerned about lining up the decimals.)

St. # 2 got the correct answer, 0.96.

11. First part of the question:

St. # 1: You can use money. I went to the store and had $4.60 and then had $5.30 and I had $9.90. (St. # 2 helped finish the story.)

Int: Is there anywhere else that I can use decimals?

St. # 1: in math class

(They couldn´t think of another example.)

Second part of the question:

St. # 1: making downpayments on a car

St. # 2: making downpayments on furniture.

St. # 1: buying car insurance.

(They could not make a story with their suggestions.)

Int: What kind of a situation could I be in that I would multiply by a decimal ?

St. # 1: You want to pay for something. You want to figure out how much 25% off is.

St. # 2: Discount

Group 4
Nov. 8,1988

GROUP 4: 1 female/1 male

NOTE: The female is supposed to be transferring into Math 13 but there is no room in the class so she will stay in math 15. She said "I know this". Alot of her answers will prove her wrong. She does not know this.

The male knew the procedures quite well.

(Student # 2 (male) arrived late.)

1. QUESTION: Which of these three numbers is the biggest: 0.62 or 0.236 or 0.4 ?

St. # 1 (female): 0.4,because there is only one place after the decimal.

Int: Which would be the smallest?

St. # 1: 0.236,because there are three places after the decimal.

2. QUESTION: Is there a number that is bigger than 4.9 but smaller than 5 ?

St. # 1: I don´t know.

The question was related to money.

St. # 1 then said that .1 was the answer.

Int: Just .1 ?

St. # 1: yes

Int: Is .1 > 4.9 and < 5 ?

St. # 1: I don´t know.

Again tried to relate this to money. She said that 4.90 could represent money, $4.90 and 5 could represent $5.

Int: Is there a number that is > 4.9 but < 5 ?

St. # 1: No. I don´t know. (She had difficulty even using money. She finally said $4.95 but once the dollar signs were ereased it meant nothing.)

THE OTHER SIDE OF THE TAPE IS GONE!!!! AAAGH!!!

NOTE: The male knew the procedures quite well, much better than the female.

The male knew how to change fractions to a decimal. I can't remember if the female could.

They were able to come up with a story for the addition part of number 11 (money) but they could not think of a situation where you would multiply decimals.

Group 5
Nov. 10,1988

GROUP 5: 1 male

1. QUESTION: Which of these three numbers is the biggest: 0.62 or 0.236 or 0.4 ?

St.: 0.4

Int: How do you know it´s 0.4 ?

St: Because you move the decimal over. I just know that it´s the biggest.

2. QUESTION: Is there a number that is bigger than 4.9 and smaller than 5 ?

St: No.

Int: What could this 4.9 look like ? What could it be if I was in a situation outside of school.

Ans: It looks as if you´re getting paid.

Int: Yes, it could represent money. What could the 5 represent ?

Ans: $5

Int: Is there anything between $4.90 and $5 ?

Ans: Sure, there´s 9 cents.

Int: What other prices could I have in between them ?

Ans: $4.91 to $4.99

Int: If I take away the dollar signs can I have a number between 4.9 and 5 ?

Ans: Yes. (He was able to make the transfer.)

3. QUESTION: CIRCLE the number nearest in size to 0.16:
0.1, 0.2, 15, 0.21, 10

St: 0.21

Int: Why is it 0.21?

St: Because 15 is lower, oh no, it doesn't have a decimal. 15 is > 0.16, .1 is >, .2 is >, 10 is >, so .21 is the closest.

We related the problem to money. .16 can represent 16 cents, .21 can represent 21 cents.

Int: How many cents are in between .16 and .21?

Ans: 5

Int: What does .1 represent?

Ans: 1 cent

Int: What would .2 represent ?

Ans: 2 cents.

He was not able to make the transfer from .16 to sixteen cents to .2 being twenty cents.

4. QUESTION: How do I subtract decimals ?

St: You move the decimal place over.

QUESTION: What would be the answer to .86-.3 ?
i) .83

(He did this:

$$\begin{array}{r} .86 \\ .03 \\ \hline .83 \end{array}$$

Int: Is .3 the same thing as .03 ?

When the problem was related to money and going to the store he came up with 56 cents.

Int: How do I show this on paper so that I get .56 for an answer?

St: Move the decimal to the right, 2 places. (Moves all decimals two places but still couldn't get the correct answer).

QUESTION: What would the answer be for 7 - .4 ?

St: Move the decimal one place to the right. (He means of the .4) Then subtract 7 from 4 is 3. It would be

.3.

5. Int:How do I multiply decimals ?

St: You line the decimals up.

QUESTION: How would I multiply .05 x 4 ?

St: Move the decimal over 2 places.

Int: Do I line up the decimals ?

St: Yes

Int: Then what do I do?

St: Then you multiply and you get the right answer.

Int: What would the answer for this question be ?

Ans: .2 (then he said .020). You multiply 4 and 5 is 20 and then multiply 4 and 0.

6. QUESTION: How would I add 6 + .32 ?

St: 6.32 (Showed a correct procedure.)

QUESTION: How would I add 5.1 + .46?

St: 5.47, then said 5.56.

Int: How did you get 5.56 ?

St: Move the decimal place over to the left and add. Oh no, I didn't move the decimal place. You line the decimal places up below each other and you add. The 5 goes down automatically, you add the one and the four and put the 6.

(He has confused his rules.)

7. QUESTION: What would be the answer to he division .028 ÷ .4 ?

St: .7 (Didn't want to do this on paper.)

Int: What did you do with the decimals?

Ans: You line them up. No, you can't do that. I just left them, and then divided. The answer would be .07 .

8. QUESTION: If I wanted to change the fraction 1/2 to a decimal what would it be ?

St: .5

QUESTION: What would the fraction 3/1000 be as a decimal?

St: I don't know that one.

Ine: We know that 1/2 = .5. If I didn't know that 1/2 was = to .5, how would I find that ?

St: Just divide a whole number in half. Say 10 is divided by 5 which would be 2. 2 x 5 is 10 so that would be half.

QUESTION: What would the fraction 1/5 be as a decimal?

St: 1.5. No that isn't right. Yeah, 1.5. ..... I don't know. I'm guessing.

Divided $1 between 5 friends. He said that each friend would get 20 cents but he couldn't relate this to the problem.

9. QUESTION: What would be the answer to this question: add 0.1 to 6.98 ?

St: 6.99

10. QUESTION: What would the answer be to this one: take away 0.1 from 1.06 ?
St: 0.96 (He did the subtraction correctly but not the addition.)

11. First part of the question:

St: I don't know.

Int: What can 4.6 represent ?

St: Money

He then made up a story with money. "John had $4.60, his father gave him $5.30, so he had $9.90."

Second part of the question:

(couldn't do this part)

Int: What could .25 represent ?

St: 25 cents, liters

(He still was unable to come up with a story.)

He knew that .25 was 25/100 and 1/4.

Appendix C

Post-test

DECIMAL KNOWLEDGE MATH 15

NAME: ____________________________________

CALCULATORS MUST NOT BE USED

1. Place the decimal in each of the following numbers so that the 4 in each case has the value of four tenths.

a) 41 b) 4318 c) 74

d) 847 e) 1874

2. a) CIRCLE the SMALLEST of these three numbers:

0.6 0.06 0.006

b) How can you tell it is the smallest? ____________________

____

____

3. Which of the following is twenty-eight thousandths ?

a) 0.028

b) 0.28

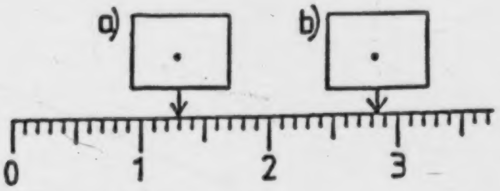
c) 28

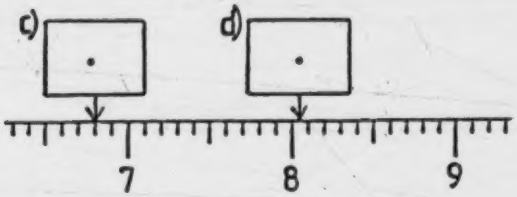
d) 28000

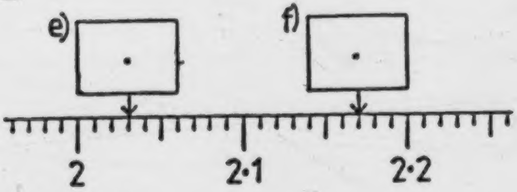
e) None of these. I think the answer is __________.

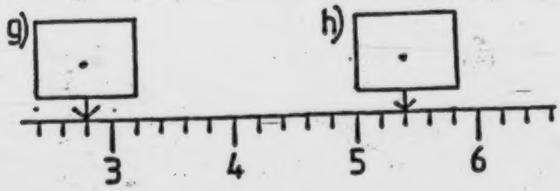
4. What number is one hundredth more than 2.468 ? ____________

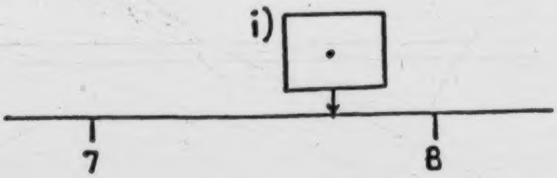
6. Read the following scales and write your answers in the boxes provided.
Give all your answers as DECIMALS.





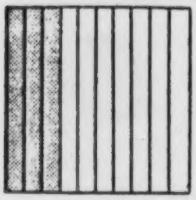






6. What decimal has been shaded in each of the following.

1.



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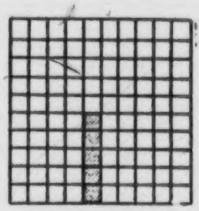
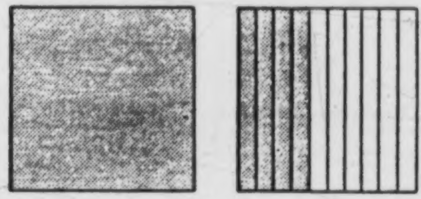


2.

____

3.



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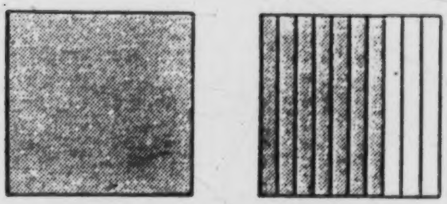
4.



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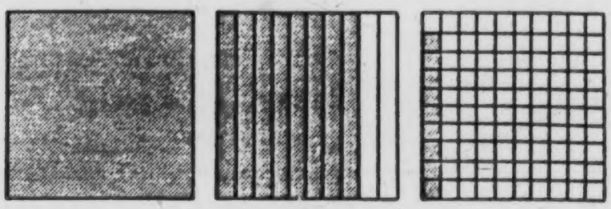

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7. CIRCLE the one that gives the BIGGER answer:

0.8 x 0.4 OR 0.8 0.4

8. Arrange the following in order of size from the LARGEST to the SMALLEST.

0.05, 0.17, 0.01, 0.15

____

9. Johnny was working in a grocery store and had to find the mass of a carton containing a dozen eggs. He knew that the mass of an average egg is 45.5 grams and that the mass of the carton is 25 grams. Johnny multiplied 45.5 g by 12 which made 546 grams and then subtracted 25 g to make 521 grams. He told his boss that the mass of the carton with a dozen eggs is 521 grams.

Is Johnny's answer correct? YES or NO

How would you have found the answer ? ____________________

____

____

10. Estimate the answer then place the decimal point in the given answer.

a) 3.36 x 5.710 = 19185600

b) 4.28 .08 = 0535

c) 0.44 x 0.76 = 0334400

11. Perform the following subtractions.

a) 7.81 - 3.5 = ________________

b) 4.79 - 1.38 = ________________

c) 43.20 - 0.16 = ________________

12. Perform the following multiplications.

a) 5.3 x 4 = ____________________

b) 8.4 x 2.1 = ____________________

c) 0.27 x 0.03 = ____________________

---

13. A stadium holds 55,000 people. The stadium is 80% full. How many people are in the stadium ?

a) More than 50,000

b) Between 47,000 and 50,000

c) Between 45,000 and 47,000

d) Less than 45,000

e) I don't know.

---

14. Sandra deposits $300 in a new bank acount which pays 9% interest. How much money will Sandra have in her bank account at the end of one year ?

a) $300

b) Between $300 and $325

c) Between $325 and $350

d) Between $350 and $400

e) I don't know.

15. Write down any number which is

a) BIGGER than 2.9 but SMALLER than 3 ____________

b) BIGGER than 4 but SMALLER than 4.1 ____________

c) BIGGER than 0.41 but SMALLER than 0.42 __________

d) BIGGER than 6.9 but SMALLER than 6.11 ___________

---

16. a) CIRCLE the number nearest in size to 0.14

0.1 0.2 15 0.21 10

b) CIRCLE the number nearest in size to 2.09

209 2.9 2.05 2.1 20.9

---

17. Perform the following additions.

a) 5.6 + 2.3 = ______________

b) 6.3 + 3.42 = ______________

c) 6.1 + .66 = ______________

---

18. Perform the following divisions.

a) 24.6 ÷ 6 = ___________________

b) 0.135 ÷ 5 = ___________________

c) 4.014 ÷ 0.03 = ________________

19. CIRCLE the CALCULATION you would need to find the answer to the following problem.

Muscle makes up 0.4 of the total body mass. Wendy has a mass of 44.8 kg. How many kilograms of muscle does Wendy have ?

a) 44.8 - 0.4 b) 44.8 ÷ 0.4

c) 44.8 + 0.4 d) 44.8 x 0.4

e) 0.4 ÷ 44.8

---

20. Which decimal is equal to the fraction 3/5 ?

a) 0.35

b) 3.5

c) 0.6

d) 0.53

e) I don't know.

---

21. What are the decimal numbers for the following fractions ?

a) 1/2 = ____________

b) 1/5 = ____________

c) 1 4/5 = ____________

d) 2/3 = ____________

Appendix D

Delayed Post-test

DECIMAL KNOWLEDGE MATH 15

NAME: ________________________________

CALCULATORS MUST NOT BE USED

---

1. a) CIRCLE the SMALLEST of these three numbers:

0.6 0.06 0.006

b) How can you tell it is the smallest? ____________________

____

____

---

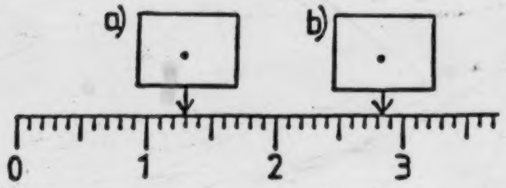
2. Which of the following is twenty-eight thousandths ?

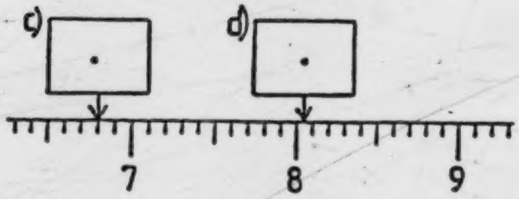
a) 0.028
b) 0.28
c) 28
d) 28000
e) None of these. I think the answer is __________.

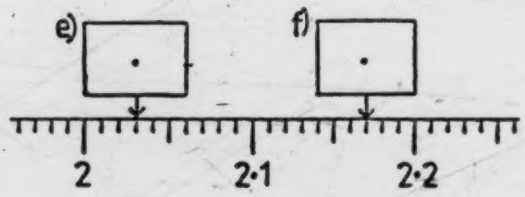
---

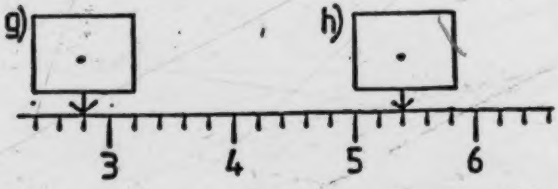
3. What number is one hundredth more than 2.468 ? __________

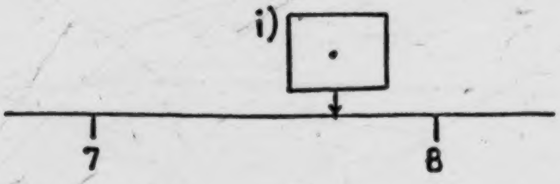
4. Read the following scales and write your answers in the boxes provided. Give all your answers as DECIMALS.





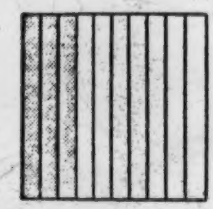






5. What decimal has been shaded in each of the following.

1.



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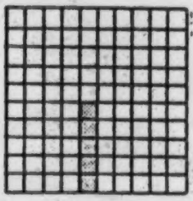
2.



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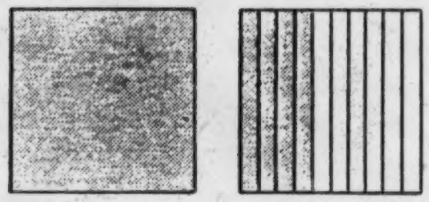
3.



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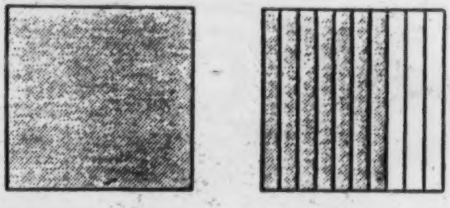
4.



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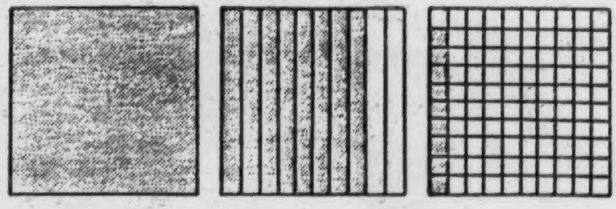
5.



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6. Estimate the answer then place the decimal point in the given answer.

a) 3.36 x 5.710 = 19185600

b) 4.28 ÷ .08 = .0535

c) 0.44 x 0.76 = 0334400

7. Perform the following subtractions.

a) 7.81 - 3.5 = ____________________

b) 4.79 - 1.38 = ____________________

c) 43.20 - 0.16 = ____________________

8. Perform the following multiplications.

a) 5.3 x 4 = ____________________

b) 8.4 x 2.1 = ____________________

c) 0.27 x 0.03 = ____________________

9. Write down any number which is

a) BIGGER than 2.9 but SMALLER than 3 ______________

b) BIGGER than 4 but SMALLER than 4.1 ______________

c) BIGGER than 0.41 but SMALLER than 0.42 ___________

d) BIGGER than 6.9 but SMALLER than 6.11 ____________

---

10. a) CIRCLE the number nearest in size to 0.14

0.1 0.2 15 0.21 10

b) CIRCLE the number nearest in size to 2.08

209 2.9 2.05 2.1 20.9

---

11 . Perform the following additions.

a) 5.6 + 2.3 = _______________

b) 6.3 + 3.42 = _______________

c) 6.1 + .66 = _______________

---

12. Perform the following divisions.

a) 24.6 ÷ 6 = ____________________

b) 0.135 ÷ 5 = ____________________

c) 4.014 ÷ 0.03 = ____________________

---

13. CIRCLE the CALCULATION you would need to find the answer to the following problem.

Muscle makes up 0.4 of the total body mass. Wendy has a mass of 44.8 kg. How many kilograms of muscle does Wendy have ?

a) 44.8 - 0.4
b) 44.8 ÷ 0.4
c) 44.8 + 0.4
d) 44.8 x 0.4
e) 0.4 ÷ 44.8

---

14. Which decimal is equal to the fraction 3/5 ?

a) 0.35
b) 3.5
c) 0.6
d) 0.53
e) I don't know.

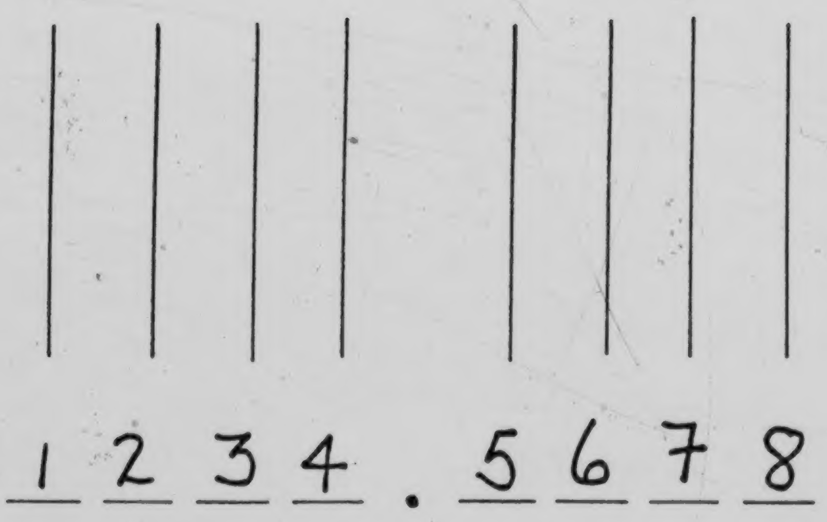
---

15. What are the decimal numbers for the following fractions ?

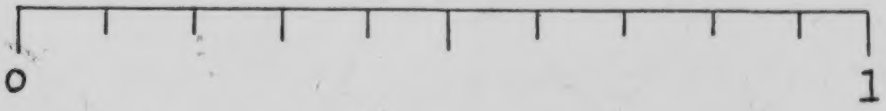
a) 1/2 = ______________
b) 1/5 = ______________
c) 1 4/5 = ______________
d) 2/3 = ______________

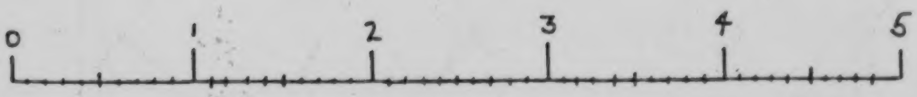
Appendix E

Place Value Worksheets



WORKSHEET E-1





WORKSHEET E-2

COMPARING DECIMALS

1. Put these names in alphabetical order.

Case
Bell
Adams
Banks
Ackroyd
Clarke
Davis
Coles
Bannister
Cook
Ferris
Cooper

2. Put these numbers in order of size, largest first.

5258
12689
126
1268
626
127
1300
1267
12

3. Put these numbers in order of size, largest first.

0.8
0.15
0.75
0.05
0.465
0.4
0.5
0.55
1

WORKSHEET E-3

COMPARING DECIMALS

1. CIRCLE the largest of these three numbers:

5.75, 5.485, 5.5

How did you decide that it was the largest?

2. Write the following numbers in ascending order (SMALLEST first).

3.87 3.9 4.07 4.5 3.49 3.785 3.6

3. In each list CIRCLE the two numbers which are equal :

a) 78 780 078 708

b) 0.150 0.15 0.015 0.105

4. CIRCLE all the numbers that are BIGGER than 0.6 :

0.12 0.8 0.45 0.536 0.4

CIRCLE all the numbers that are SMALLER than 0.75 :

0.6 0.125 0.096 0.8 0.106

5. Write down, if possible, any number that is BIGGER than the first, but SMALLER than the second :

a) 3.5 < __________ < 3.7

b) 3.9 < __________ < 3.11

c) 4.1 < __________ < 4.2

d) 5 < __________ < 5.1

e) 5.1 < __________ < 5.11

WORKSHEET E-4

DECIMALS

1. CIRCLE the larger of the two numbers: 10.9 or 10.75.

How do you know it is bigger?

2. Write the following numbers in decreasing order (BIGGEST first).

1.86   1.9   2.07   2.5   1.842   2.1   1.7756

3. CIRCLE all the numbers that are BIGGER than 0.4 :

0.15   0.09   0.5   0.104   0.3

CIRCLE all the numbers that are SMALLER than 0.45 :

0.6   0.125   0.373   0.3   0.085

4. Write down any number that is BIGGER than the first, but SMALLER than the second.

example: $6 < 8.5 < 10$

a) $6.2 <$ ______ < 6.5

b) $0.9 <$ ______ < 1

c) $1 <$ ______ < 1.1

5. In each of the following, CIRCLE the two numbers which are equal :

a) 098 , 908 , 980 , 98

b) 0.32 , 0.032 , 0.302 , 0.320

c) 06.5 , 60.5 , 6.05 , 6.50

WORKSHEET E-5

PLACE VALUE

1. Write the number that is one tenth greater.

a) 2.5 ________________ b) 1.9 ________________

c) 25.7 ________________ d) 0.68 ________________

e) 125.2 ________________ f) 0.7 ________________

2. Write the number that is one hundredth greater.

a) 8.75 ________________ b) 0.93 ________________

c) 1.085 ________________ d) 3.346 ________________

e) 17.591 ________________ f) 0.083 ________________

3. Write the number that is one thousandth less.

a) 5.238 ________________ b) 6.088 ________________

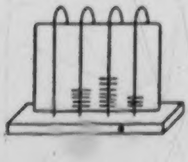
c) 0.13 ________________ d) 4.959 ________________

e) 6.781 ________________ f) 15.325 ________________

WORKSHEET E-6

DECIMALS AND THE ABACUS

46.3 can be shown on an abacus

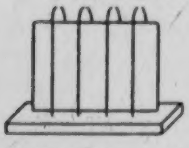


The number 46.3 is written below in expanded form.

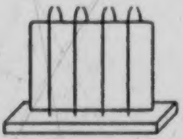
40 + 6 + 0.3

= 4 tens + 6 ones + 3 tenths
= (4x10) + (6x1) + (3x0.1)
= (4x10) + (6x1) + (3x1/10)

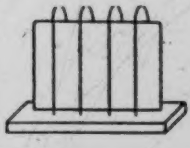
A.
Show each of the following expanded forms on the abacus.



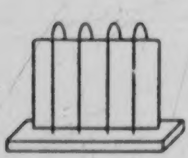
1. 30 + 3 + 0.1



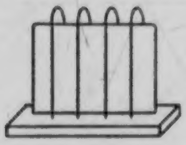
2. 40 + 1 + 0.9



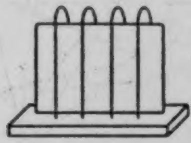
3. 4 + 0.1



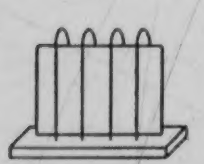
4. 50 + 7 + 0.5



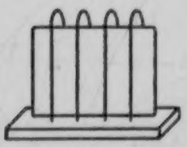
5. (5 x 4) + (3 x 1) + (2 x 0.1)



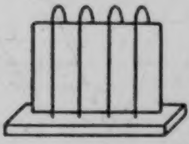
6. (4 x 10) + (4 x 1) + (2 x 0.1)



7. (3 x 10) + (5 x 1) + (4 x 0.1)



8. (3 x 10) + (5 x 1) + (2 x $\frac{1}{10}$)



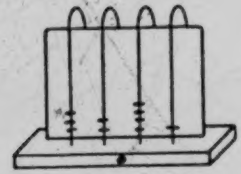
9. (6 x 10) + (4 x 1) + (3 x $\frac{1}{10}$)

WORKSHEET E-7

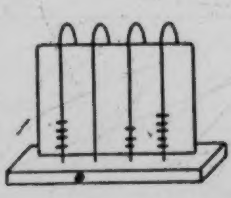
NUMBERS ON THE ABACUS

What number is represented on each abacus?
The first two are done for you.

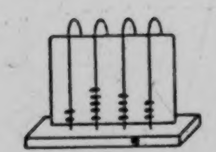
1. 32.41



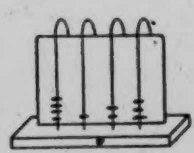
2. 4.035



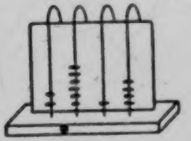
3.



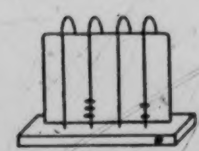
4.



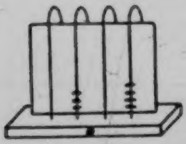
5.



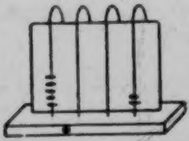
6.



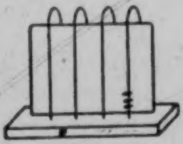
7.



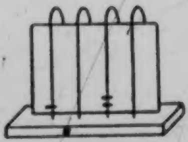
8.



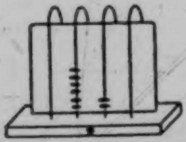
9.



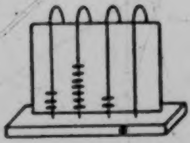
10.



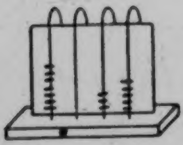
11.



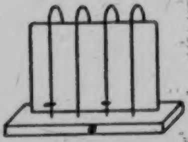
12.



13.

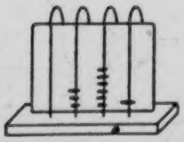


14.

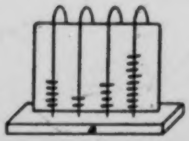


WORKSHEET E-8

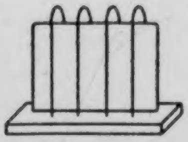
On each abacus, show the numeral written under it.
The first two are done for you.



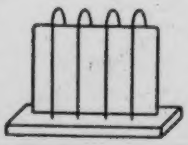
1. 36.1



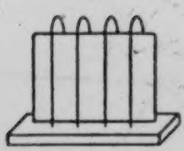
2. 52.49



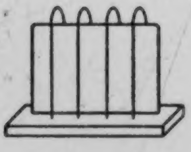
3. 1.634



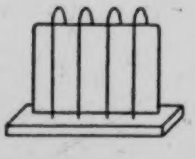
4. 0.542



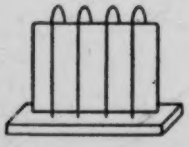
5. 286



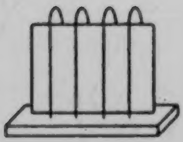
6. 2.86



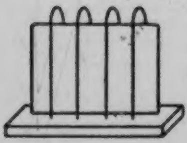
7. 0.026



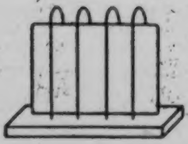
8. 0.206



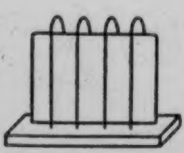
9. 10.1



10. 1.01



11. 0.131



12. 36

WORKSHEET E-9

THE FEWEST NUMBER OF COINS

What is the fewest number of coins or bills needed to make each of the following amounts? Record your answers in the spaces provided. The first one is done for you.

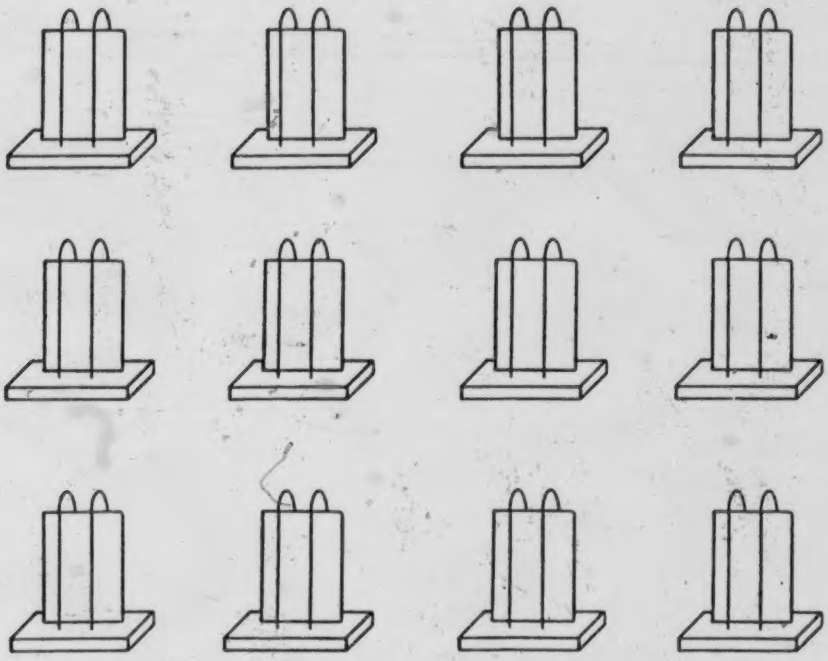
A.

	$1.00	$0.50	$0.25	$0.10	$0.05	$0.01	Total Number
$0.07					1	11	3
$0.09							
$0.13							
$0.28							
$0.59							
$0.75							
$2.78							
$3.61							
$5.51							
$6.03							

B.

	$5.00	$2.00	$1.00	$0.50	$0.25	$0.10	$0.1	Total Number
$2.98								
$3.96								
$5.97								
$6.97								
$9.98								
$7.98								
$8.92								
$10.01								
$12.24								
$10.99								

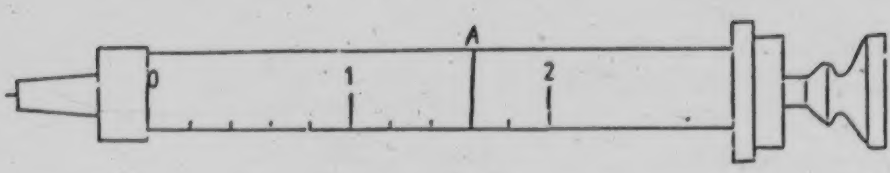
WORKSHEET E-10

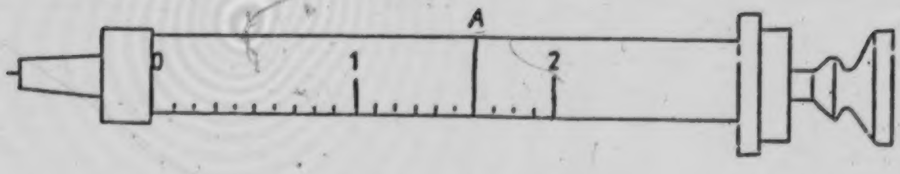


WORKSHEET E-11

Appendix F

Scale Reading Worksheets



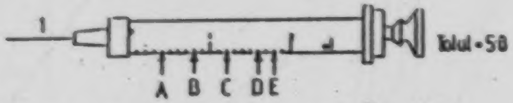


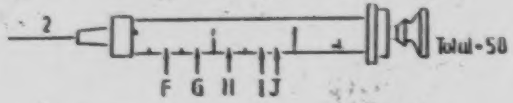
WORKSHEET F-1

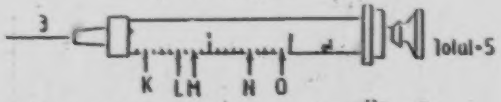
GIVING INJECTIONS

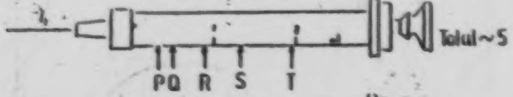
Imagine you are a nurse or doctor. Read each of the following doses.

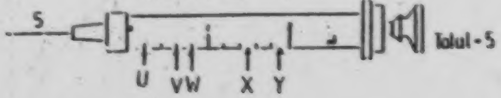
After each question, add up your five answers. You should get the totals shown on the right hand side.

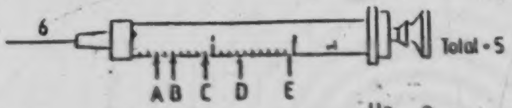

1
A B C DE
Total = 50

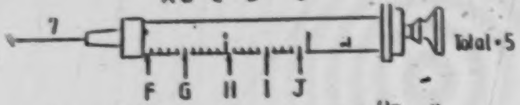

2
F G H IJ
Total = 50

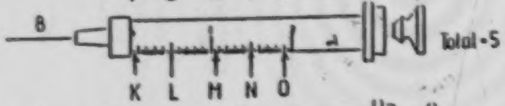

3
K LM N O
Total = 5

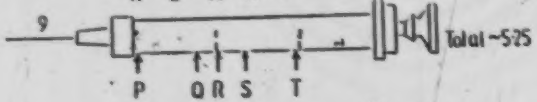

4
PQ R S T
Total ~ 5

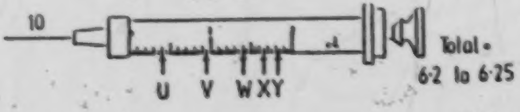

5
U VW X Y
Total = 5


6
AB C D E
Total = 5


7
F G H I J
Total = 5


8
K L M N O
Total = 5

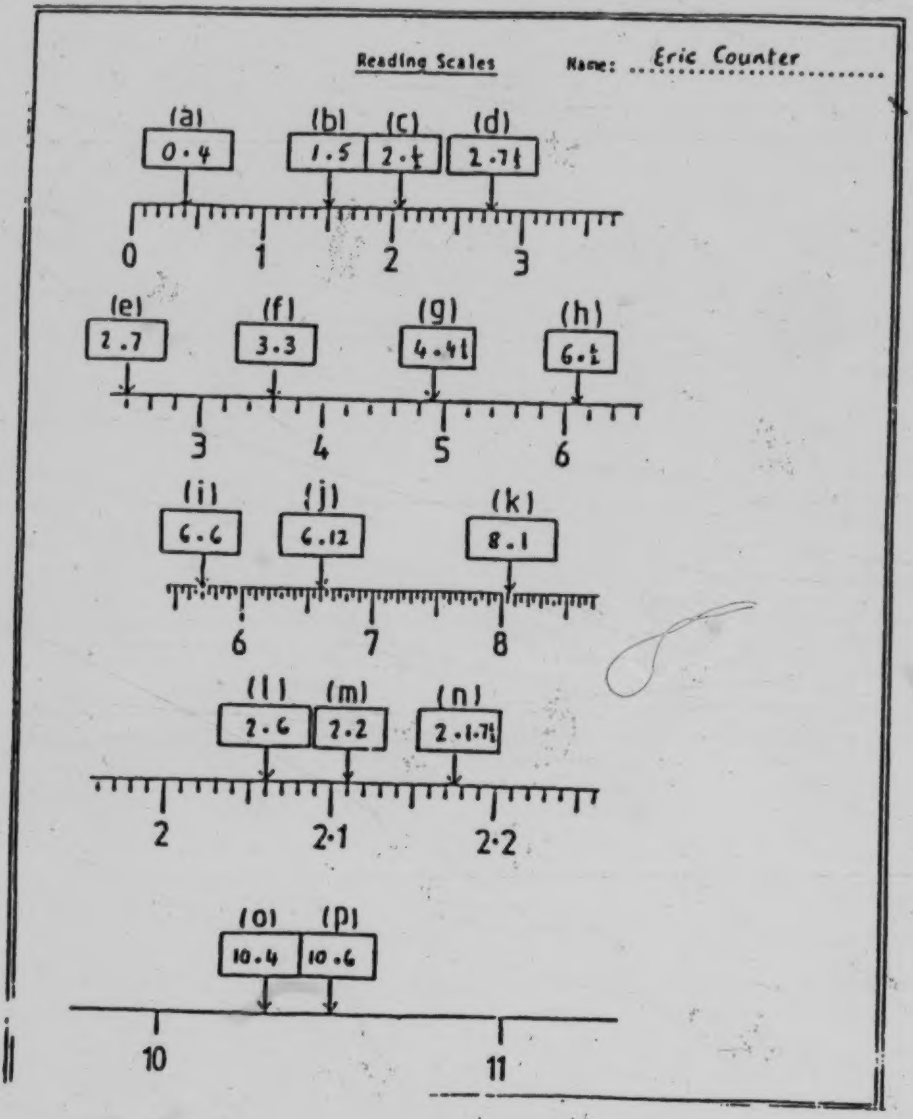

9
P QR S T
Total ~ 5·25


10
U V WXY
Total = 6·2 to 6·25

WORKSHEET F-2

MARKING HOMEWORK

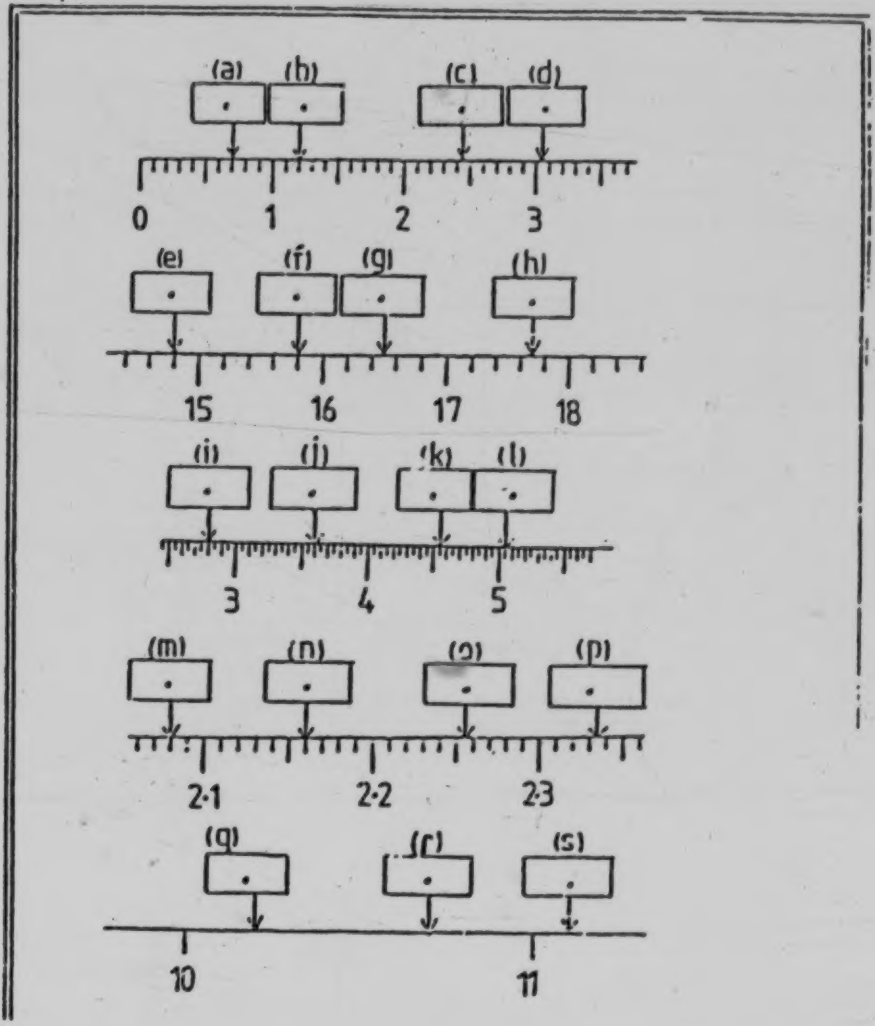
The following worksheet was given to Eric Counter for homework. Mark the work correcting all the mistakes.
Can you see what Eric is doing wrong?


Reading Scales
Name: Eric Counter
(a) 0·4
(b) 1.5
(c) 2·½
(d) 2·7½
0
1
2
3
(e) 2.7
(f) 3.3
(g) 4·4½
(h) 6·½
3
4
5
6
(i) 6.6
(j) 6.12
(k) 8.1
6
7
8
(l) 2·6
(m) 2.2
(n) 2·1·7½
2
2·1
2·2
(o) 10.4
(p) 10.6
10
11

WORKSHEET F-3

FURTHER PRACTICE SHEET

Read the following scales and write your answers in the spaces provided.


(a)
(b)
(c)
(d)
0
1
2
3
(e)
(f)
(g)
(h)
15
16
17
18
(i)
(j)
(k)
(l)
3
4
5
(m)
(n)
(o)
(p)
2·1
2·2
2·3
(q)
(r)
(s)
10
11

WORKSHEET F-4

Appendix G

Fraction-to-Decimal Translation Worksheets

FRACTIONS

1/2 3/5 1/10 1/4 1/5
90/100 5/5 1/3 4/5 3/4

DECIMALS

0.1 1.0 0.5 0.90 0.6
0.25 0.20 0.75 0.8 0.33

PERCENTS

90% 50% 60% 100% 20%
33.3% 10% 25% 80% 75%

INSTRUCTIONS:

For each of the above fractions there is an equivalent decimal and percent. Fill the chart below with the fractions in the list and the decimal and percent that is equal to each fraction.

FRACTIONS	DECIMALS	PERCENTS

WORKSHEET G-1

FRACTIONS/DECIMALS/PERCENTS

INSTRUCTIONS:

Work each problem and locate the answer in the answer section at the bottom of the page. Place the answer in the blank space at the front of each problem.

PROBLEMS:

1. ____________ 23 hundredths

2. ____________ the ratio of 13 to 100

3. ____________ 0.45 as a percent

4. ____________ 1/5 as a percent

5. ____________ 30% of 610

6. ____________ 3 cents is what percent of a dollar

7. ____________ $10 is what percent of $2

8. ____________ 80% as a decimal

9. ____________ 15% as a fraction

10. ____________ 3/12 as a decimal

11. ____________ 1 3/4 as a decimal

12. ____________ 15% of 2,000

ANSWERS:

13/100
20%
500%
183
0.23
3/20
0.8
45%
3%
0.25
1.75
300

WORKSHEET G-2

FRACTIONS/DECIMALS/PERCENTS

1. Express each of the following percents as a decimal.

a) 15% b) 38% c) 71%

2. Express each of the following decimals as a percent.

a) 0.17 b) 0.63 c)0.88 d) 2.36

3. Express each of the following fractions as a percent.

a) 29/100 b) 46/100

c) 123/100 d) 2/10

4. A movie theatre has seating for 1239 people. If 82% of the seats are occupied, how many people are at the theatre ?

5. A stereo costs $578.5 .
a) The sales tax is 7%. How much sales tax would be added to the price of the stereo?

b) What is the total cost of the stereo?

c) How much change will you get from $600.00 ?

6. Malinda earns $178 per week plus 3.5% commission on everything she sells. How much will she earn if she sells $1600.00 worth of goods?

7. A valuable stamp was worth $180.00 . Its value has increased 510% of the original value. What is the value of the stamp today ?

WORKSHEET G-3

FRACTIONS/DECIMALS/PERCENTS

The following rectangle contains fractions, decimals and percents. Take each number and place it in the correct column in the second rectangle.

$\frac{1}{3}$ $\frac{25}{100}$ $\frac{2}{10}$ $\frac{3}{8}$ 17%
50% 18 100% $\frac{29}{100}$ $\frac{23}{100}$
$\frac{3}{4} - \frac{1}{2}$ $\frac{3}{10}$ .18 25% .09
.25 5% $\frac{5}{8} - \frac{1}{2}$ 2.1 $\frac{25}{10}$ .5 $\frac{1}{5}$

$< \frac{1}{4}$	$\frac{1}{4}$	$> \frac{1}{4}$

Make the following true, using

= means "equals"
< means "less than"
> means "greater than"

$\frac{1}{2}$ ☐ 50% $\frac{1}{4}$ ☐ 30% 10% ☐ $\frac{10}{100}$
$\frac{1}{3}$ ☐ 25% $\frac{2}{5}$ ☐ 40% 75% ☐ .8
90% ☐ $\frac{90}{100}$ 100% ☐ $\frac{100}{100}$ 100% ☐ 1
125% ☐ $1\frac{1}{4}$ 150% ☐ $1\frac{3}{4}$ 200% ☐ $1\frac{1}{2}$
23% ☐ $\frac{1}{4}$ $\frac{3}{4}$ ☐ 72% 1% ☐ .1

WORKSHEET G-4

Appendix H

Operations Worksheets

SEQUENCES

Complete the following sequences:

1. Adding on 0.2´s.

0.2, 0.4, 0.6, _____, _____, _____, _____, _____, _____, _____,
_____, _____, _____,

2. Adding on 0.3´s.

0.3, 0.6, 0.9, _____, _____, _____, _____, _____, _____, _____,
_____, _____, _____,

3. Adding on 0.4´s.

0.4, 0.8, _____, _____, _____, _____, _____, _____, _____, _____,
_____.

4. Adding on 0.5´s.

0.5, _____, _____, _____, _____, _____.

5. Adding on 0.25´s.

0.25, _____, _____, _____, _____, _____, _____, _____.

6. Adding on 0.05´s.

0.05, _____, _____, _____, _____, _____, _____, _____, _____,
_____, _____, _____, _____, _____, _____, _____, _____, _____, _____.

7. Adding on 0.15´s.

0.15, _____, _____, _____, _____, _____, _____, _____, _____,
_____, _____, _____.

8. Adding on 0.125

0.125, _____, _____, _____, _____, _____, _____, _____,
_____, _____, _____, _____, _____, _____, _____.

WORKSHEET H-1

SEQUENCES

Complete the following sequences:

1. Adding on 0.3's.

8.2, 8.5, ______, ______, ______, ______, ______, ______, ______, ______.

2. Subtracting 0.3's.

6.4, 6.1, ______, ______, ______, ______, ______, ______, ______

3. Adding on 0.1's.

3.51, ______, ______, ______, ______, ______, ______, ______.

4. Subtracting 0.05's.

100.35, ______, ______, ______, ______, ______,
______, ______.

5. Halving

3.2, 1.6, ______, ______, ______, ______, ______, ______, ______.

6. Subtracting 0.04's.

1.28, ______, ______, ______, ______, ______, ______, ______.

7. Subtracting 0.09's.

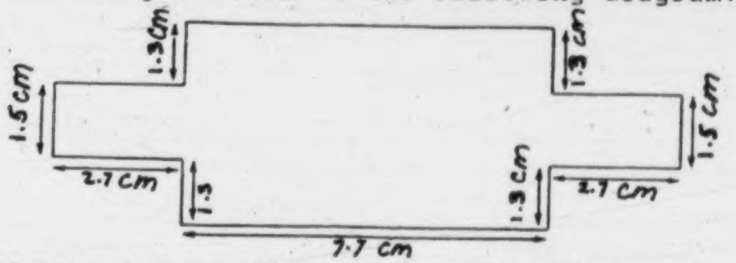
56.36, ______, ______, ______, ______, ______, ______.

WORKSHEET H-2

WORKSHEET H-3

DECIMAL PROBLEMS

1. Find the perimeter of the following diagram.

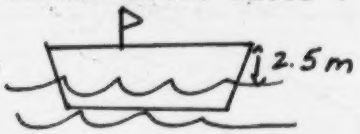


2. Richard had an odometer on his bicycle. He found that a one-way trip to the nearest 7-Eleven was 0.2 km. How far did he travel if he made three round trips to the same 7-Eleven?

3. An engineer has a piece of wire 0.825 cm long. He wants to cut it into six equal parts. How long will each piece be ?

4. Bonnie Jones and her family heated 342.75 gallons of hot water one month. This was 56.47 gallons less than the previous month. How many gallons did they heat in the previous month ?

5. The gunwale of a boat is 2.5 meters above water. If the tide rises 1 meter, how much of the gunwale of the boat is now above water ?



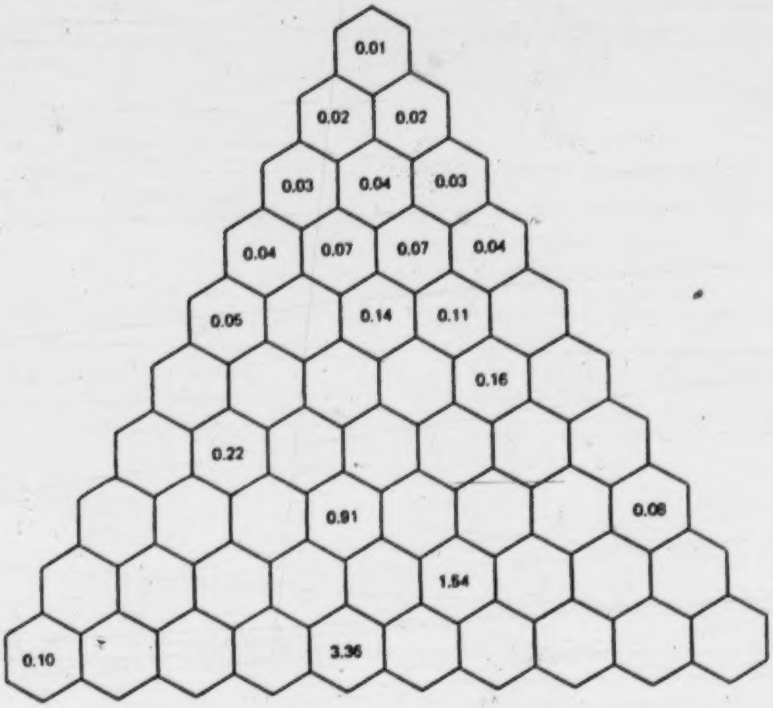
6. The flood stage for a river is 12.46 meters. One day the river was at 8.24 meters. If the river kept rising at 0.4 meters per hour, in how many hours would it reach flood stage ?

7. You took three of your friends to the best restaurant in town for dinner one night. The bill came to $384.79. The service was great and you wish to leave a 15% tip. How much should you leave for a tip ?

COMPLETE THE PYRAMID

Study the following pyramid to see how the numbers are obtained.

Complete the pyramid by supplying the missing numbers.


0.01
0.02
0.02
0.03
0.04
0.03
0.04
0.07
0.07
0.04
0.05
0.14
0.11
0.16
0.22
0.91
0.08
1.54
0.10
3.36

2. If this pyramid were enlarged, what would be the first number in the 30th row? ____________________

3. What would be the last number in the 105th row? ____________________

WORKSHEET H-4

ADD AND SUBTRACT

1. Which person would win each of the following events?

a) Sprinting:
Tyrus, 11.4 seconds or Stecher, 11.07 seconds

b) Women's High Jump:
Reskova, 1.82 m; Meyforth, 1.92 m, or
Achermann, 1.93 meters.

c) Women's Long Jump:
Rosendahl, 6.67 metres or Voigt, 6.59 meters.

2. In the 1972 Olympic Games, Annelie Ehrhardt of East Germany won the 100-meter hurdles in a time of 12.59 seconds. In the 1980 Olympic Games, Vera Komisova of USSR won the same event in a time of 12.56 seconds. Which competitor had the better time?

3. Arrange the numbers from smallest to largest:

0.0016 0.160 0.1601 0.0061 0.0611

4. In one year, the production of one kind of home computer was 15486 units. The following year 18486 units were produced. What was the increase in production ?

WORKSHEET H-5

5. During this year the Ultimate Health Club increased its membership by 72 persons. If there were 786 members last year, how many are there this year ?

6. The sum of two numbers is 56.94. One of the numbers is 17.35. What is the other number?

7. Susan bought a paperback book for $3.95. She gave the clerk a $10 bill. How much did she get in change?

MULTIPLY AND DIVIDE

1. Jovani made a long-distance call from Antigonish, N.S. to Moncton, N.B., on Sunday at 9 a.m. He talked for 7 minutes. What did the call cost him if the rate was $0.58 per minute?

2. Martha wants to make $55 this week. Her part time job pays $2.75 per hour. How many hours must she work?

3. While hauling a truck load of oranges, the transport truck was involved in an accident and spilled 2604 oranges. How many dozen oranges were spilled?

4. For a championship football game between the Wildcats and the Reddogs the admission charge was $2.25. The price included the bus ride to and from the stadium. How much did it cost 347 people to attend?

5. A football stadium has seating for 68300 people. If 84% of the seats are occupied, how many people are at the football game?

6. A class of 18 Math 15 students is planning a field trip to New York. The total cost of the trip is $5760.00. If the school gives $900 towards the cost of the trip, how much will each student have to pay to go on the trip?

7. An apple has 28.4 mg of vitamin C. A banana has 11.6 mg of vitamin C.

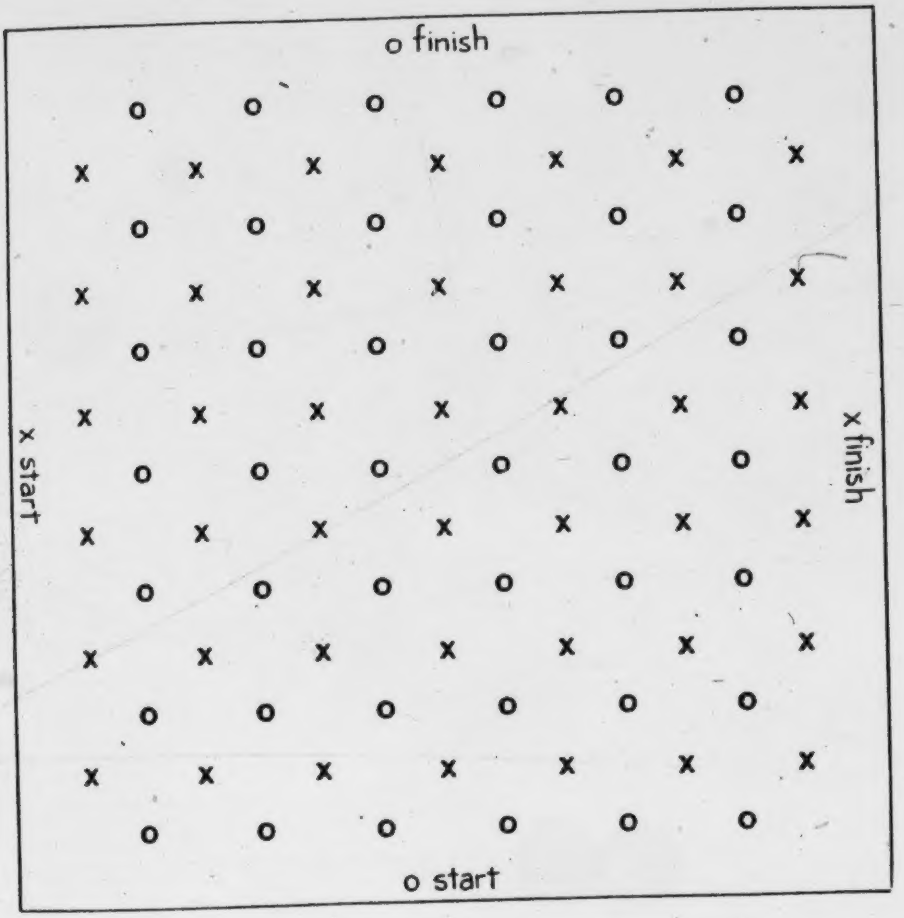
   a) How much more vitamin C is there in an apple than in a banana ?

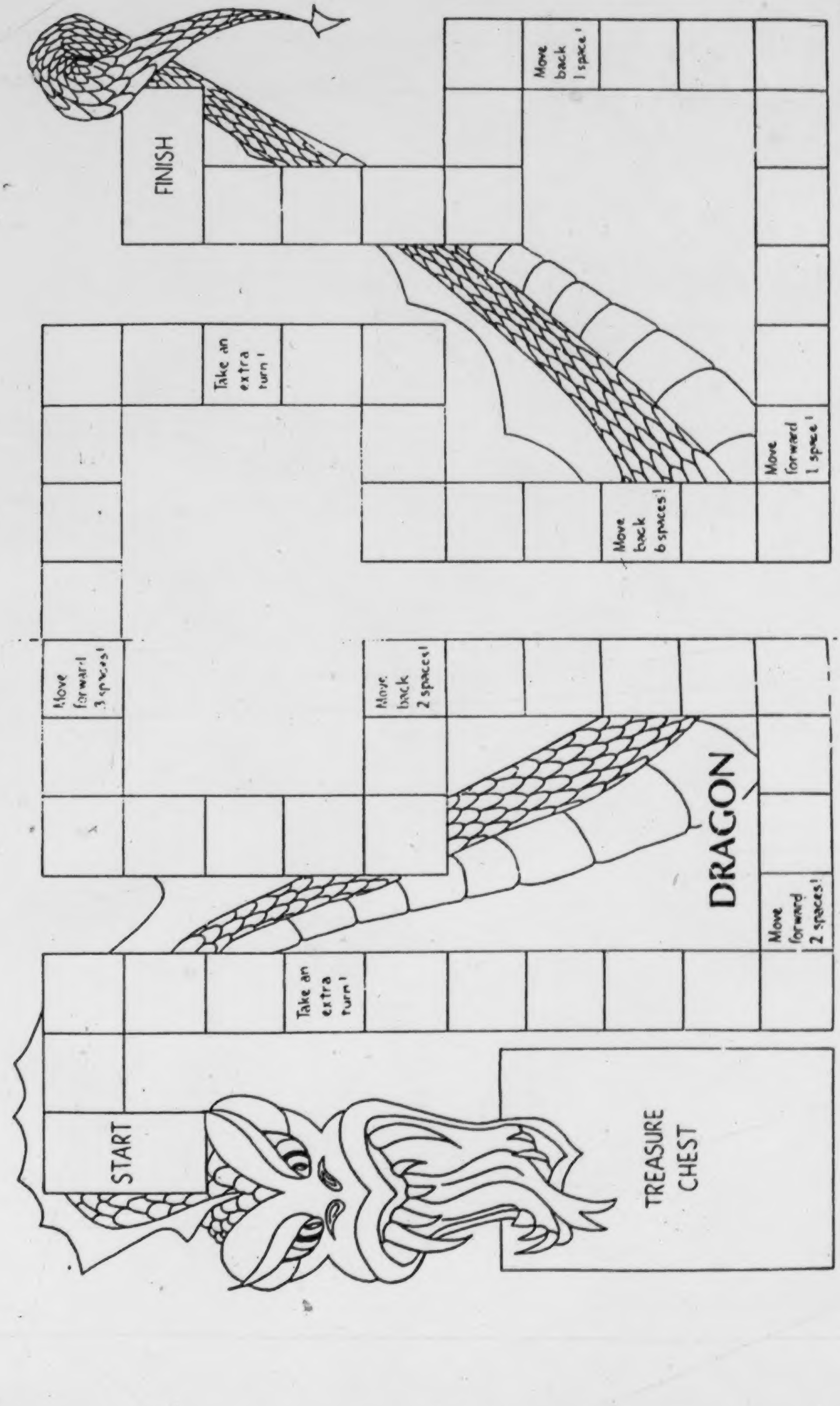
   b) How much vitamin C would you get if you ate 2 bananas and half an apple ?

Appendix I

Games

BARRICADE


o finish
x start
x finish
o start


START
TREASURE CHEST
Take an extra turn!
Move forward 2 spaces!
DRAGON
Move back 2 spaces!
Move forward 3 spaces!
Take an extra turn!
Move back 6 spaces!
Move forward 1 space!
Move back 1 space!
FINISH

Appendix J

Percentage of Correct Responses

Table 5:3
Percentage of Correct Responses to Items 1 and 10

Question	Pretest N = 29	Post-test N = 24	Delayed Post-test N = 22
1. a)	28%	79%	86%
b)	7%	17%	18%
10. a)	7%	25%	23%
b)	10%	17%	23%

Table 5:4
Percentage of Correct Responses to Items 2 and 3

Question	Pretest N = 29	Post-test N = 24	Delayed Post-test N = 22
2	34%	54%	59%
3	66%	54%	50%

Table 5:5
Percentage of Correct Responses to Items 4 and 5*

Question		Pretest N = 29	Post-test N = 24	Delayed Post-test N = 22
4	a	86%	83%	96%
	b	17%	29%	23%
	c	62%	83%	82%
	d	14%	42%	36%
	e	17%	29%	23%
	f	34%	54%	50%
	g	10%	21%	36%
	h	10%	21%	41%
	i	66%	71%	82%
5	a	79%		
	b	62%		
	c	21%		
	d	10%		
5	1		54%	59%
	2		33%	55%
	3		50%	55%
	4		46%	55%
	5		38%	50%

* Item 5 was not the same on the three tests.

Table 5:6
Percentage of Correct Responses to Items 14 and 15

Question		Pretest N = 29	Post-test N = 24	Delayed Post-test N = 22
14		10%	38%	36%
15	a	45%	54%	68%
	b	31%	33%	55%
	c	no comparable item		
	d	17%	21%	41%

Table 5:7
Percentage of Correct Responses to Items 7, 8, 11 and 12

Question		Pretest N = 29	Post-test N = 24	Delayed Post-test N = 22
7	a	34%	79%	95%
	b	83%	92%	100%
	c	79%	83%	95%
8	a	76%	79%	91%
	b	62%	67%	77%
	c	45%	54%	82%
11	a	93%	88%	91%
	b	86%	83%	95%
	c	76%	83%	86%
12	a	52%	71%	59%
	b	21%	21%	41%
	c	38%	33%	55%

Appendix K

Results of Pretest/Post-test/Delayed Results

QUESTION	PRETEST N = 29	POST-TEST N = 8	DELAYED N = 6
#2.	8	7	4
#3.	10	4	4
#4.	19	3	3
#5. a)	25	7	5
b)	5	1	1
c)	18	7	3
d)	4	3	2
e)	5	4	2
f)	10	4	4
g)	3	2	3
h)	3	2	3
i)	19	5	5
#6. a)	23		
b)	18		
c)	6		
d)	3		
#6. 1)		4	2
2)		2	2
3)		4	2
4)		4	2
5)		2	2
#10. a)	4	1	1
b)	3	error	1
c)	3	0	1
#11. a)b)	24	8	6
b)c)	23	7	5
c)a)	10	7	6
#12. a)b)	18	6	6
b)a)	22	7	5
c)c)	13	6	6

QUESTION	PRETEST N = 29	POST-TEST N = 8	DELAYED N = 6
#15. a)	6	3	4
b)	4	3	4
c)d)	4	0	0
d)c)	4	3	4
#16. a)	2	1	0
b)	3	1	1
#17. a)	27	8	6
b)	25	7	6
c)	22	7	6
#18. a)	15	6	5
b)c)	11	2	3
c)d)	6	1	2
#19.	11	1	0
#20.	3	3	3
#21. a)	13	5	5
b)d)	5	3	3
c)b)	9	4	5